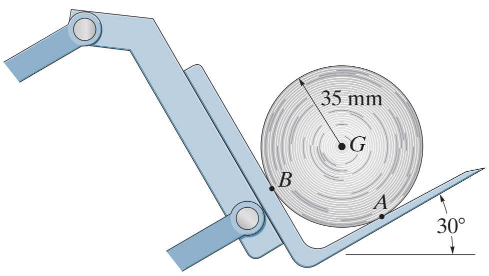
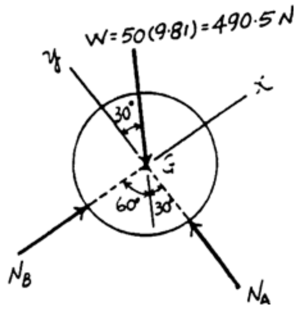
•5–1. Draw the free-body diagram of the 50-kg paper roll which has a center of mass at G and rests on the smooth blade of the paper hauler. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



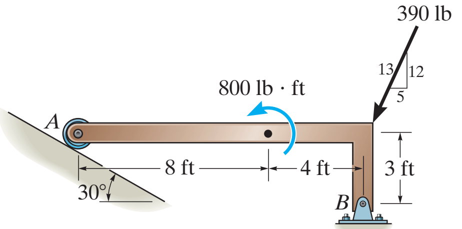
The Significance of Each Force:

W is the effect of gravity (weight) on the paper roll.

N_A and N_B are the smooth blade reactions on the paper roll.

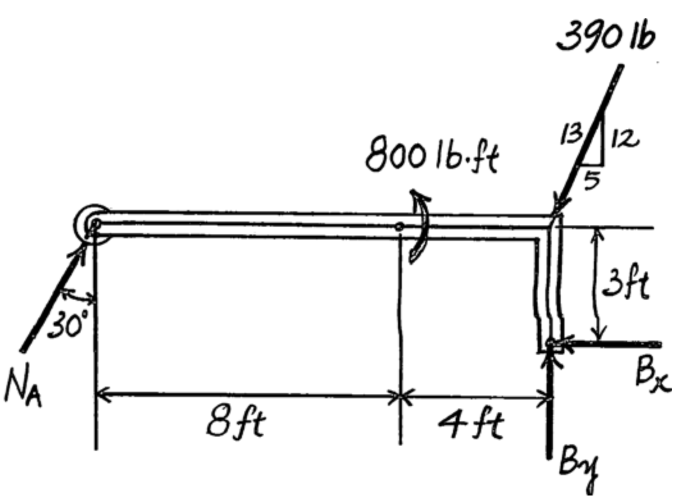


5–2. Draw the free-body diagram of member AB, which is supported by a roller at A and a pin at B. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)

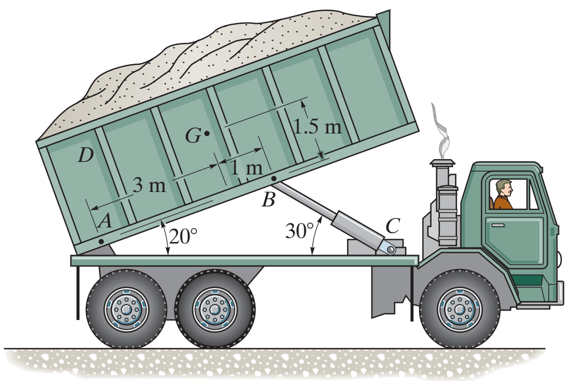


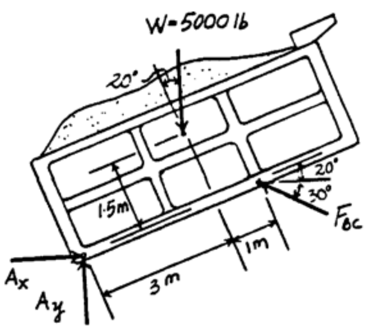
N_A force of plane on roller.

B_x, B_y force of pin on member.



5–3. Draw the free-body diagram of the dumpster D of the truck, which has a weight of 5000 lb and a center of gravity at G. It is supported by a pin at A and a pin-connected hydraulic cylinder BC (short link). Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)





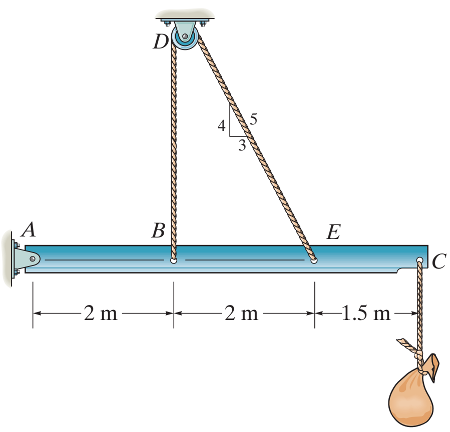
The Significance of Each Force :

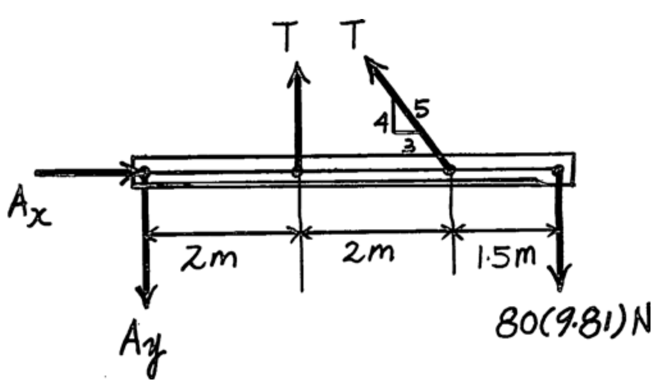
W is the effect of gravity (weight) on the dumpster.

A_y and A_x are the pin A reactions on the dumpster.

F_{BC} is the hydraulic cylinder BC reaction on the dumpster.

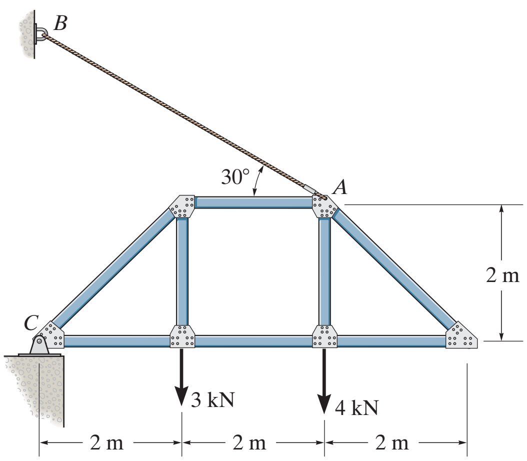
***5–4.** Draw the free-body diagram of the beam which supports the 80-kg load and is supported by the pin at A and a cable which wraps around the pulley at D. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)

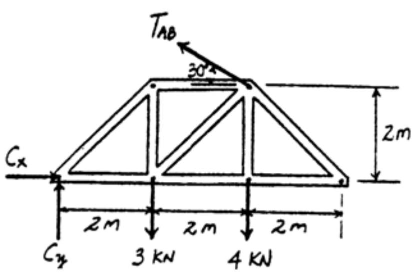




T force of cable on beam.
A_x, A_y force of pin on beam.
80(9.81)N force of cable on beam.

•5–5. Draw the free-body diagram of the truss that is supported by the cable AB and pin C. Explain the significance of each force acting on the diagram. (See Fig. 5–7b.)





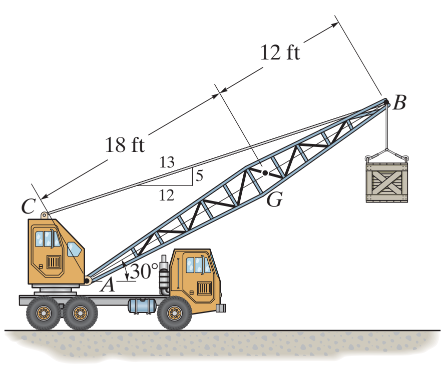
The Significance of Each Force :

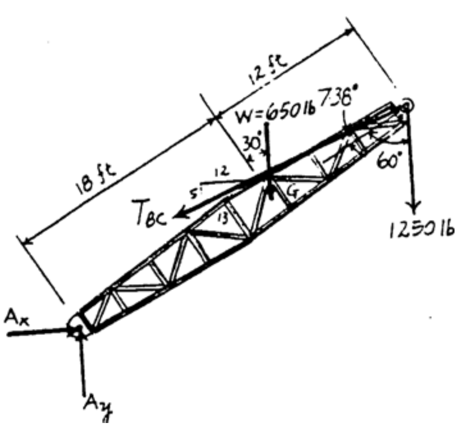
C_y and C_x are the pin C reactions on the truss.

T_{AB} is the cable AB tension on the truss.

3 kN and 4 kN force are the effect of external applied forces on the truss.

5–6. Draw the free-body diagram of the crane boom AB which has a weight of 650 lb and center of gravity at G. The boom is supported by a pin at A and cable BC. The load of 1250 lb is suspended from a cable attached at B. Explain the significance of each force acting on the diagram. (See Fig. 5–7b.)





The Significance of Each Force :

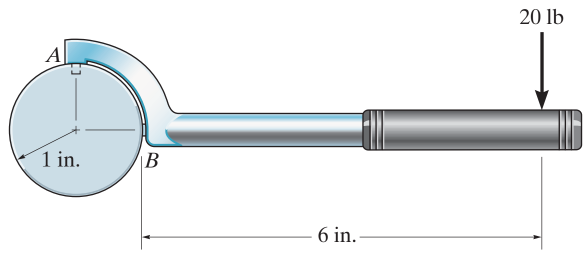
W is the effect of gravity (weight) on the boom.

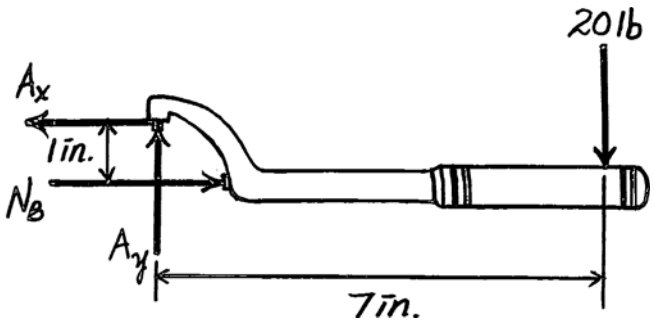
A_y and A_x are the pin A reactions on the boom.

T_{BC} is the cable BC force reactions on the boom.

1250 lb force is the suspended load reaction on the boom.

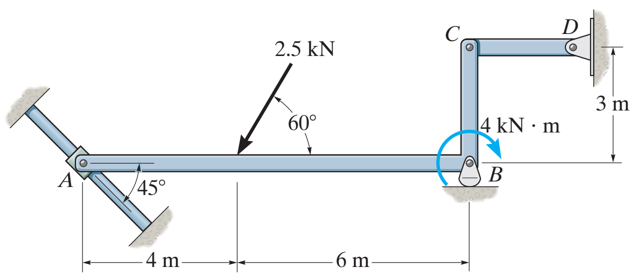
5–7. Draw the free-body diagram of the "spanner wrench" subjected to the 20-lb force. The support at A can be considered a pin, and the surface of contact at B is smooth. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)



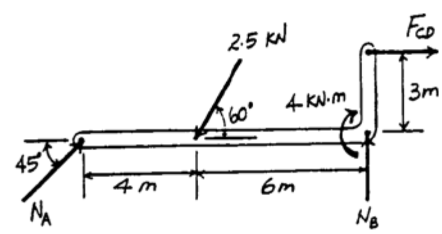


A_x, A_y, N_B force of cylinder on wrench.

***5–8.** Draw the free-body diagram of member ABC which is supported by a smooth collar at A, roller at B, and short link CD. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



The Significance of Each Force :



N_A is the smooth collar reaction on member ABC.

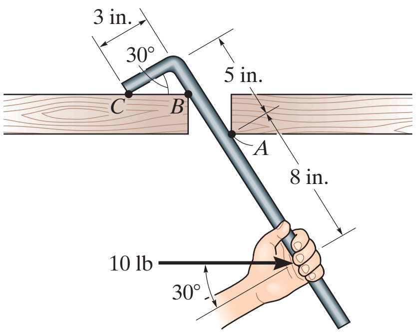
N_B is the roller support B reaction on member ABC.

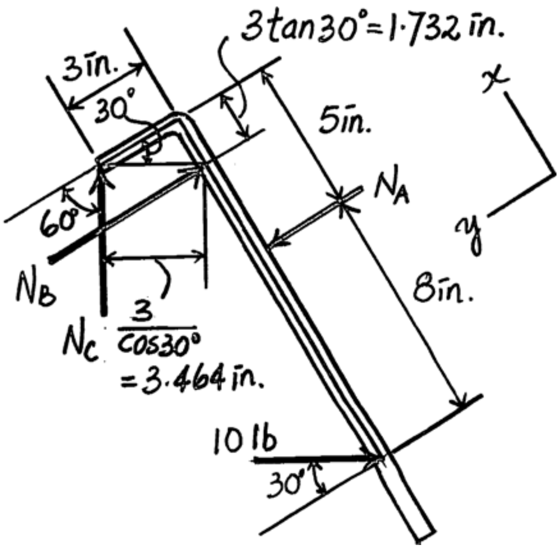
F_{CD} is the short link reaction on member ABC.

2.5 kN is the effect of external applied force on member ABC.

4 kN · m is the effect of external applied couple moment on member ABC.

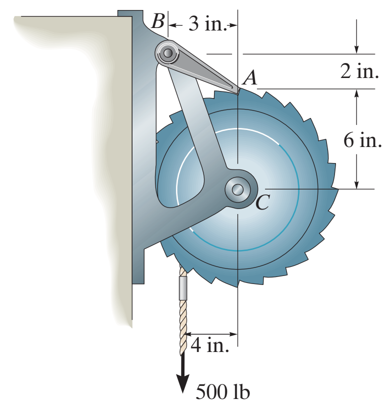
•5–9. Draw the free-body diagram of the bar, which has a negligible thickness and smooth points of contact at A, B, and C. Explain the significance of each force on the diagram. (See Fig. 5–7b.)

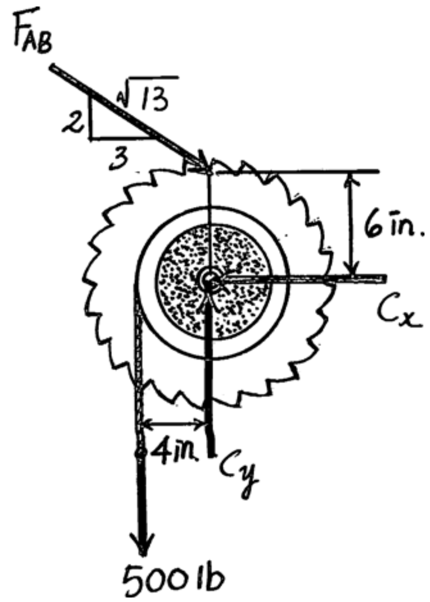




N_A, N_B, N_C force of wood on bar.
10 lb force of hand on bar.

5–10. Draw the free-body diagram of the winch, which consists of a drum of radius 4 in. It is pin-connected at its center C, and at its outer rim is a ratchet gear having a mean radius of 6 in. The pawl AB serves as a two-force member (short link) and prevents the drum from rotating. Explain the significance of each force on the diagram. (See Fig. 5–7b.)





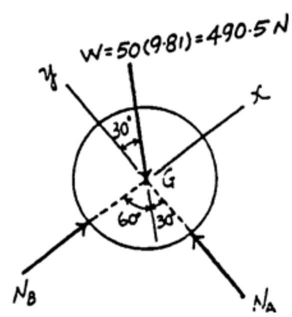
C_x, C_y force of pin on drum.
F_{AB} force of pawl on drum gear.
500 lb force of cable on drum.

5–11. Determine the normal reactions at A and B in Prob. 5–1.

Equations of Equilibrium : By setting up the x and y axes in the manner shown, one can obtain the direct solution for N_A and N_B.

$\nearrow + \Sigma F_x = 0;\quad N_B - 490.5 \sin 30° = 0 \quad N_B = 245 \text{ N}$ **Ans**

$\nwarrow + \Sigma F_y = 0;\quad N_A - 490.5 \cos 30° = 0 \quad N_A = 425 \text{ N}$ **Ans**



***5–12.** Determine the tension in the cord and the horizontal and vertical components of reaction at support *A* of the beam in Prob. 5–4.

$\circlearrowleft + \Sigma M_A = 0; \quad T(2) + T\left(\frac{4}{5}\right)(4) - 80(9.81)(5.5) = 0$

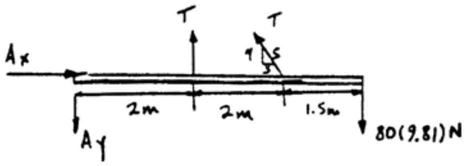
$T = 830.1 \text{ N} = 830 \text{ N}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 830.1\left(\frac{3}{5}\right) = 0$

$A_x = 498 \text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad -A_y + 830.1 + 830.1\left(\frac{4}{5}\right) - 80(9.81) = 0$

$A_y = 709 \text{ N}$ **Ans**



•5–13. Determine the horizontal and vertical components of reaction at *C* and the tension in the cable *AB* for the truss in Prob. 5–5.

Equations of Equilibrium : The tension in the cable can be obtained directly by summing moments about point *C*.

$\circlearrowleft + \Sigma M_C = 0; \quad T_{AB}\cos 30°(2) + T_{AB}\sin 30°(4) - 3(2) - 4(4) = 0$

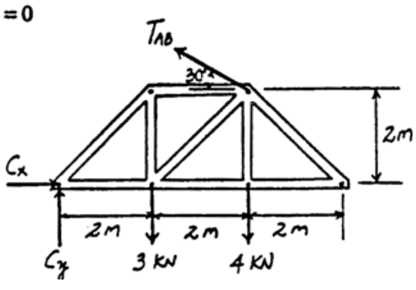
$T_{AB} = 5.89 \text{ kN}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 5.89\cos 30° = 0$

$C_x = 5.11 \text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad C_y + 5.89\sin 30° - 3 - 4 = 0$

$C_y = 4.05 \text{ kN}$ **Ans**



5–14. Determine the horizontal and vertical components of reaction at A and the tension in cable BC on the boom in Prob. 5–6.

Equations of Equilibrium : The force in cable BC can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \quad T_{BC} \sin 7.380°(30) - 650\cos 30°(18) - 1250\sin 60°(30) = 0$$

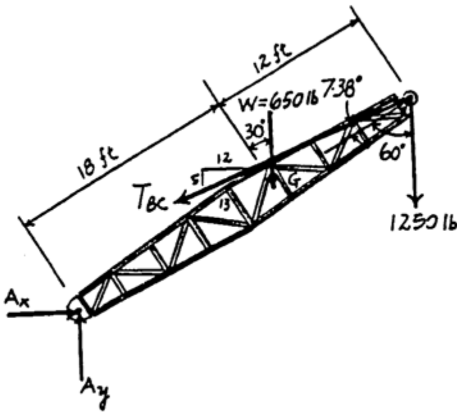
$$T_{BC} = 11056.9 \text{ lb} = 11.1 \text{ kip} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 11056.9\left(\frac{12}{13}\right) = 0$$

$$A_x = 10206.4 \text{ lb} = 10.2 \text{ kip} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 650 - 1250 - 11056.9\left(\frac{5}{13}\right) = 0$$

$$A_y = 6152.7 \text{ lb} = 6.15 \text{ kip} \qquad \textbf{Ans}$$



5–15. Determine the horizontal and vertical components of reaction at A and the normal reaction at B on the spanner wrench in Prob. 5–7.

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(1) - 20(7) = 0$$

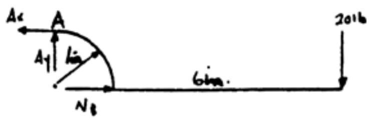
$$N_B = 140 \text{ lb} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + 140 = 0$$

$$A_x = 140 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 20 = 0$$

$$A_y = 20 \text{ lb} \qquad \textbf{Ans}$$



***5–16.** Determine the normal reactions at A and B and the force in link CD acting on the member in Prob. 5–8.

Equations of Equilibrium : The normal reaction N_A can be obtained directly by summing moments about point C.

$$\circlearrowleft + \Sigma M_C = 0; \quad 2.5\sin 60°(6) - 2.5\cos 60°(3) - 4 + N_A\cos 45°(3) - N_A \sin 45°(10) = 0$$

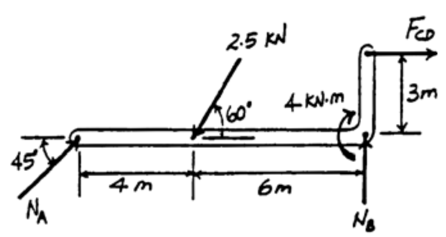
$$N_A = 1.059 \text{ kN} = 1.06 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 1.059\cos 45° - 2.5\cos 60° + F_{CD} = 0$$

$$F_{CD} = 0.501 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad N_B + 1.059\sin 45° - 2.5\sin 60° = 0$$

$$N_B = 1.42 \text{ kN} \qquad \textbf{Ans}$$



•5–17. Determine the normal reactions at the points of contact at A, B, and C of the bar in Prob. 5–9.

$+\nwarrow \Sigma F_x = 0;$ $N_C \sin 60° - 10 \sin 30° = 0$

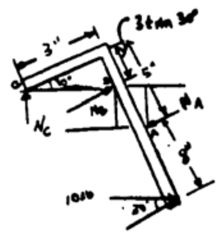
$N_C = 5.77 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_B = 0;$ $10 \cos 30°(13 - 1.732) - N_A(5 - 1.732) - 5.77(3.464) = 0$

$N_A = 23.7 \text{ lb}$ **Ans**

$+\nearrow \Sigma F_y = 0;$ $N_B + 5.77 \cos 60° + 10 \cos 30° - 23.7 = 0$

$N_B = 12.2 \text{ lb}$ **Ans**



5–18. Determine the horizontal and vertical components of reaction at pin C and the force in the pawl of the winch in Prob. 5–10.

$\circlearrowleft + \Sigma M_C = 0;$ $F_{AB}\left(\frac{3}{\sqrt{13}}\right)6 - 500(4) = 0$

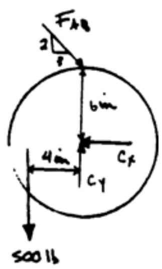
$F_{AB} = 400.6 \text{ lb} = 401 \text{ lb}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $-C_x + 400.6\left(\frac{3}{\sqrt{13}}\right) = 0$

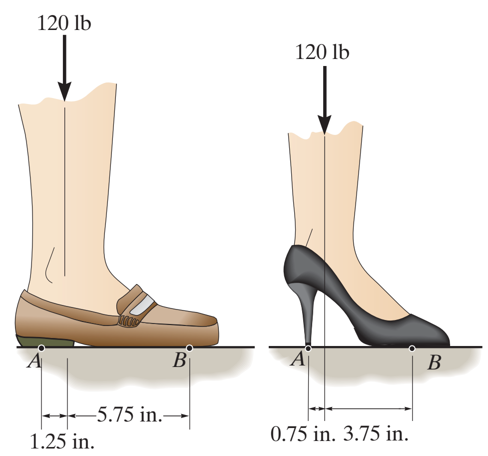
$C_x = 333 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $-500 + C_y - 400.6\left(\frac{2}{\sqrt{13}}\right) = 0$

$C_y = 722 \text{ lb}$ **Ans**



5–19. Compare the force exerted on the toe and heel of a 120-lb woman when she is wearing regular shoes and stiletto heels. Assume all her weight is placed on one foot and the reactions occur at points *A* and *B* as shown.

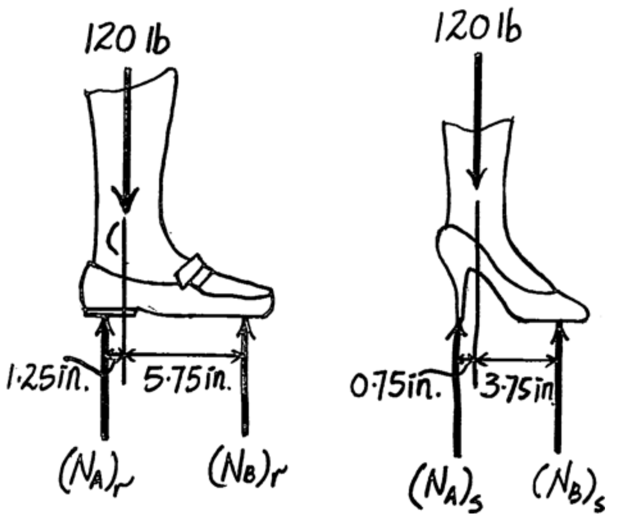


$\circlearrowleft + \Sigma M_B = 0;\quad 120(5.75) - (N_A)_r(7) = 0$

$(N_A)_r = 98.6 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad (N_B)_r + 98.6 - 120 = 0$

$(N_B)_r = 21.4 \text{ lb}$ **Ans.**



Stiletto heal shoe,

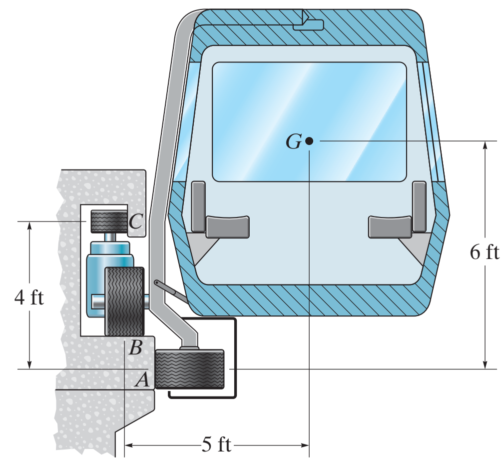
$\circlearrowleft + \Sigma M_B = 0;\quad 120(3.75) - (N_A)_s(4.5) = 0$

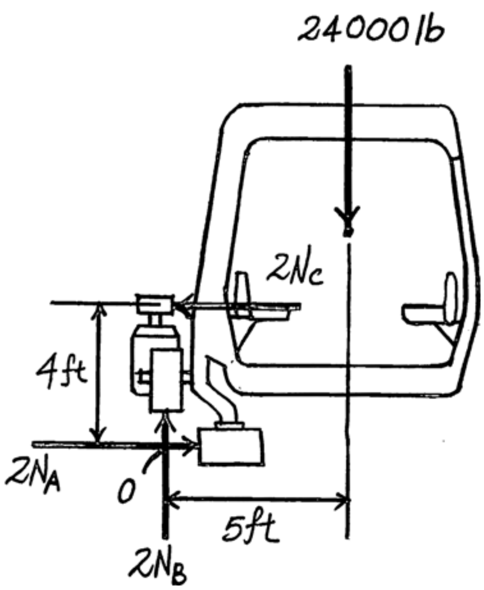
$(N_A)_s = 100 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad (N_B)_s + 100 - 120 = 0$

$(N_B)_s = 20 \text{ lb}$ **Ans.**

***5–20.** The train car has a weight of 24 000 lb and a center of gravity at G. It is suspended from its front and rear on the track by six tires located at A, B, and C. Determine the normal reactions on these tires if the track is assumed to be a smooth surface and an equal portion of the load is supported at both the front and rear tires.





$\circlearrowleft + \Sigma M_O = 0;\quad (2N_C)(4) - 24\,000(5) = 0$

$N_C = 15\,000\text{ lb} = 15\text{ kip}$ **Ans**

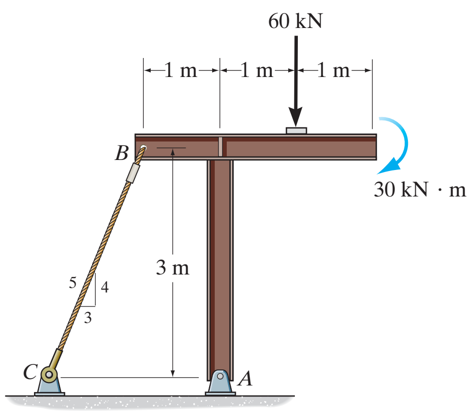
$\xrightarrow{+} \Sigma F_x = 0;\quad 2N_A - 2(15) = 0$

$N_A = 15\text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 2N_B - 24\,000 = 0$

$N_B = 12\text{ kip}$ **Ans**

•5–21. Determine the horizontal and vertical components of reaction at the pin A and the tension developed in cable BC used to support the steel frame.



Equations of Equilibrium: From the free - body diagram of the frame, Fig. *a*, the tension T of cable BC can be obtained by writing the moment equation of equilibrium about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T\left(\frac{3}{5}\right)(3) + T\left(\frac{4}{5}\right)(1) - 60(1) - 30 = 0$$

$$T = 34.62 \text{ kN} = 34.62 \text{ kN} \qquad \textbf{Ans.}$$

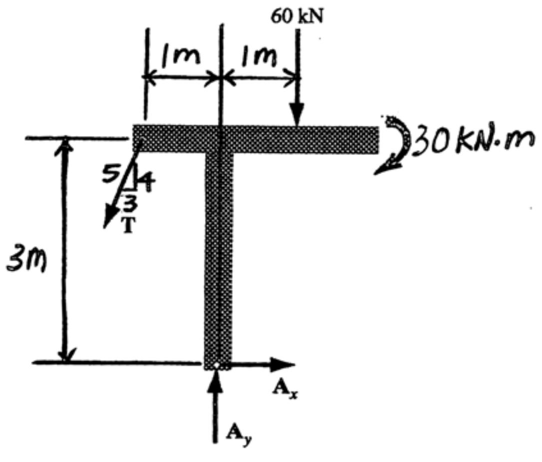
Using this result and writing the force equations of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 34.62\left(\frac{3}{5}\right) = 0$$

$$A_x = 20.77 \text{ kN} = 20.8 \text{ kN} \qquad \textbf{Ans.}$$

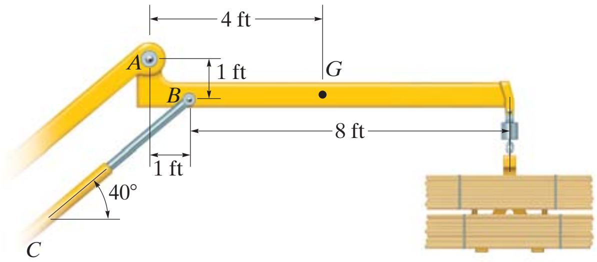
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 60 - 34.62\left(\frac{4}{5}\right) = 0$$

$$A_y = 87.69 \text{ kN} = 87.7 \text{ kN} \qquad \textbf{Ans.}$$



(a)

5–22. The articulated crane boom has a weight of 125 lb and center of gravity at G. If it supports a load of 600 lb, determine the force acting at the pin A and the force in the hydraulic cylinder BC when the boom is in the position shown.



$\circlearrowleft + \Sigma M_A = 0; \quad F_B \cos 40^\circ (1) + F_B \sin 40^\circ (1) - 125(4) - 600(9) = 0$

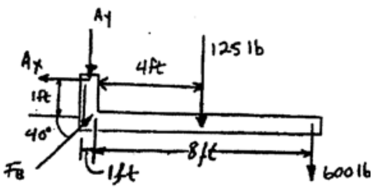
$F_B = 4188 \text{ lb} = 4.19 \text{ kip} \qquad \textbf{Ans}$

$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + 4188 \cos 40^\circ = 0$

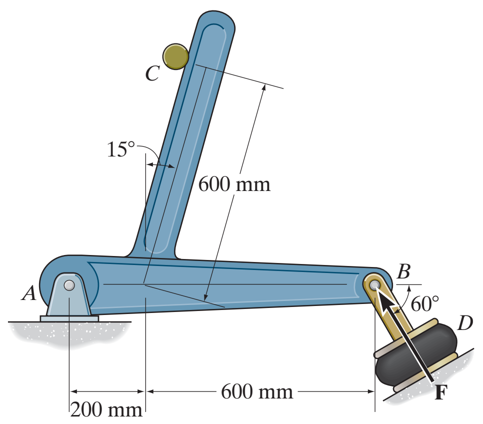
$A_x = 3208 \text{ lb} = 3.21 \text{ kip} \qquad \textbf{Ans}$

$+\uparrow \Sigma F_y = 0; \quad -A_y + 4188 \sin 40^\circ - 600 - 125 = 0$

$A_y = 1.97 \text{ kip} \qquad \textbf{Ans}$



5–23. The airstroke actuator at D is used to apply a force of $F = 200$ N on the member at B. Determine the horizontal and vertical components of reaction at the pin A and the force of the smooth shaft at C on the member.



Equations of Equilibrium: From the free - body diagram of member ABC, Fig. a, N_C can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 200 \sin 60° (800) - N_C(600 + 200 \sin 15°) = 0$

$N_C = 212.60 \text{ N} = 213 \text{ N}$ **Ans.**

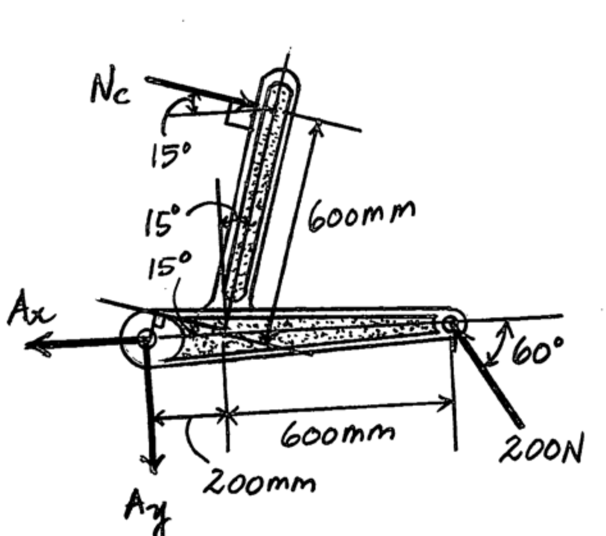
Using this result and writing the force equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -A_x + 212.60 \cos 15° - 200 \cos 60° = 0$

$A_x = 105 \text{ N}$ **Ans.**

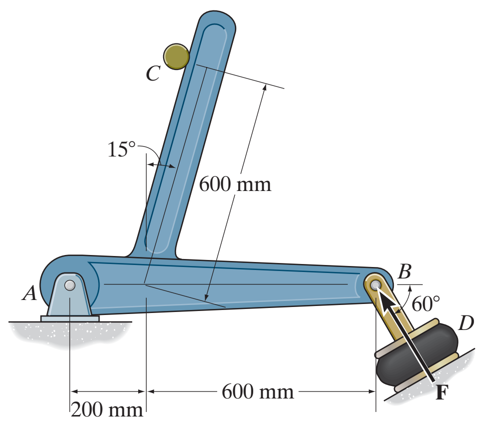
$+\uparrow \Sigma F_y = 0;$ $\quad -A_y - 212.60 \sin 15° + 200 \sin 60° = 0$

$A_y = 118 \text{ N}$ **Ans.**



(a)

***5–24.** The airstroke actuator at D is used to apply a force of **F** on the member at B. The normal reaction of the smooth shaft at C on the member is 300 N. Determine the magnitude of **F** and the horizontal and vertical components of reaction at pin A.



Equations of Equilibrium: From the free - body diagram of member ABC, Fig. a, force F can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft +\Sigma M_A = 0;\qquad F\sin 60°(800) - 300(600 + 200\sin 15°) = 0$

$F = 282.22\text{ N} = 282\text{ N}$ **Ans.**

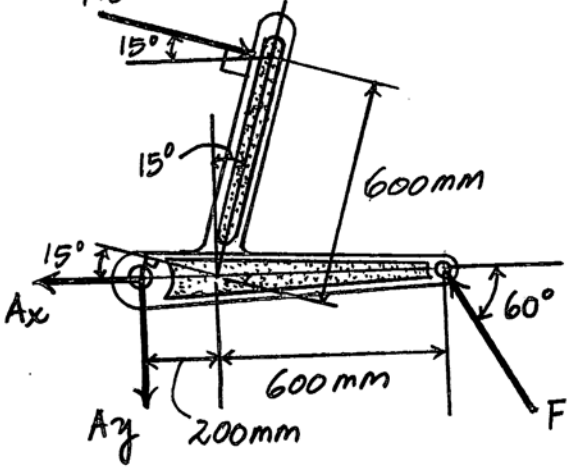
Using this result and writing the force equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad -A_x + 300\cos 15° - 282.22\cos 60° = 0$

$A_x = 149\text{ N}$ **Ans.**

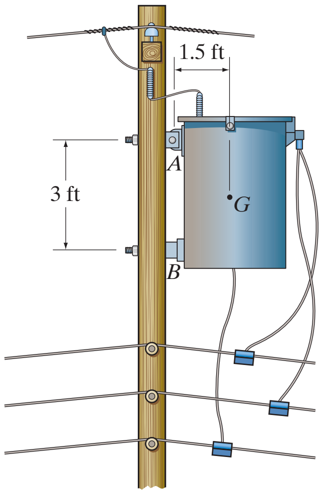
$+\uparrow \Sigma F_y = 0;\qquad -A_y - 300\sin 15° + 282.22\sin 60° = 0$

$A_y = 167\text{ N}$ **Ans.**



(a)

•**5–25.** The 300-lb electrical transformer with center of gravity at G is supported by a pin at A and a smooth pad at B. Determine the horizontal and vertical components of reaction at the pin A and the reaction of the pad B on the transformer.



Equations of Equilibrium: From the free - body diagram of the transformer, Fig. a, N_B and A_y can be obtained by writing the moment equation of equilibrium about point A and the force equation of equilibrium along the y axis.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B(3) - 300(1.5) = 0$

$N_B = 150 \text{ lb}$ **Ans.**

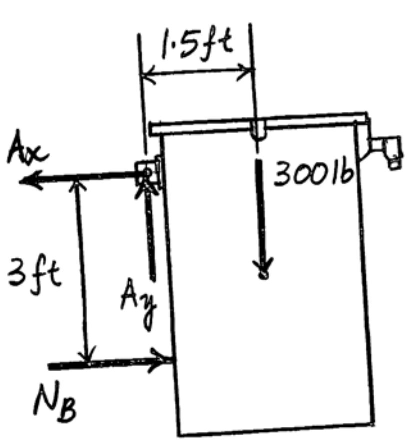
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 300 = 0$

$A_y = 300 \text{ lb}$ **Ans.**

Using the result $N_B = 150$ lb and writing the force equation of equilibrium along the x axis,

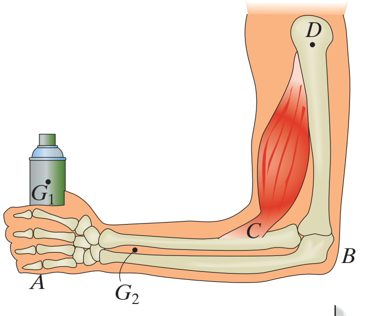
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 150 - A_x = 0$

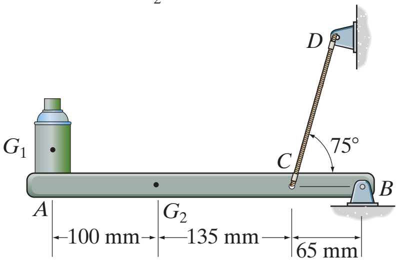
$A_x = 150 \text{ lb}$ **Ans.**



(a)

5–26. A skeletal diagram of a hand holding a load is shown in the upper figure. If the load and the forearm have masses of 2 kg and 1.2 kg, respectively, and their centers of mass are located at G_1 and G_2, determine the force developed in the biceps *CD* and the horizontal and vertical components of reaction at the elbow joint *B*. The forearm supporting system can be modeled as the structural system shown in the lower figure.





Equations of Equilibrium: From the free - body diagram of the structural system, Fig. *a*, F_{CD} can be obtained by writing the moment equation of equilibrium about point *B*.

$$\circlearrowleft +\Sigma M_B = 0; \quad 2(9.81)(100 + 135 + 65) + 1.2(9.81)(135 + 65) - F_{CD}\sin 75^\circ(65) = 0$$

$$F_{CD} = 131.25\text{ N} = 131\text{ N} \qquad \textbf{Ans.}$$

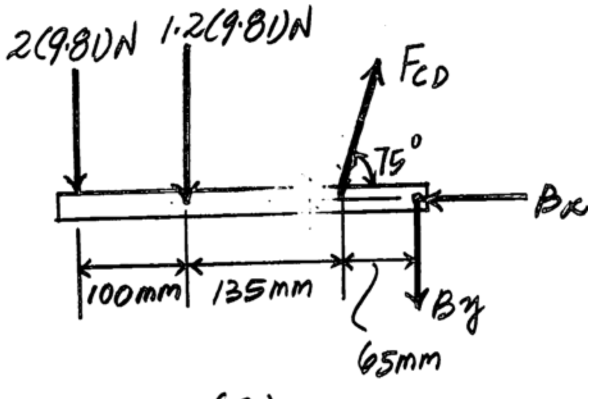
Using the above result and writing the force equations of equilibrium along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 131.25\cos 75^\circ - B_x = 0$$

$$B_x = 33.97\text{ N} = 34.0\text{ N} \qquad \textbf{Ans.}$$

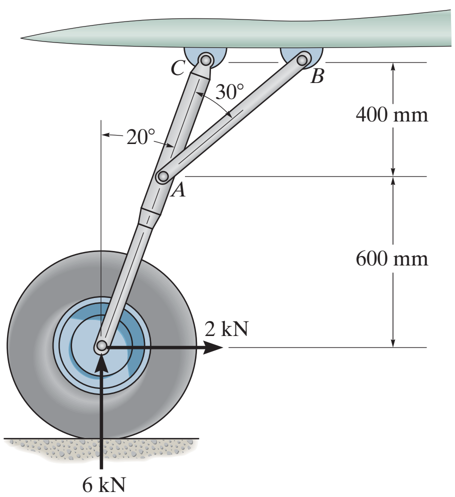
$$+\uparrow \Sigma F_y = 0; \quad 131.25\sin 75^\circ - 2(9.81) - 1.2(9.81) - B_y = 0$$

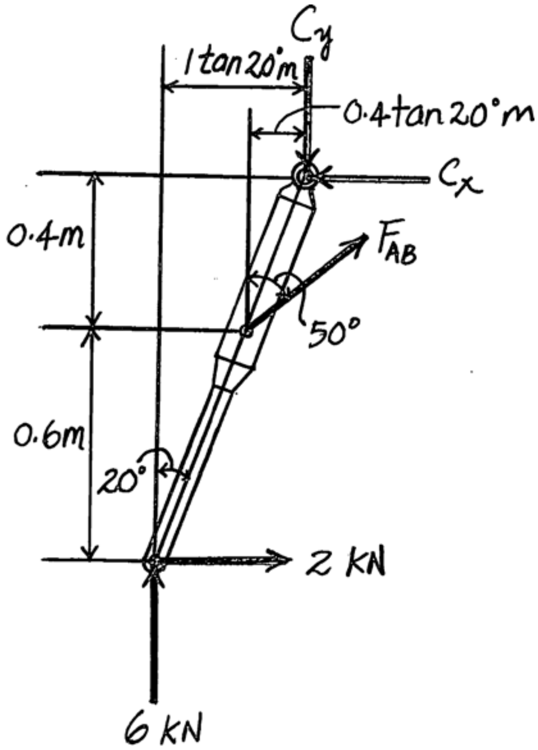
$$B_y = 95.38\text{ N} = 95.4\text{ N} \qquad \textbf{Ans.}$$



(a)

5–27. As an airplane's brakes are applied, the nose wheel exerts two forces on the end of the landing gear as shown. Determine the horizontal and vertical components of reaction at the pin C and the force in strut AB.





Equations of Equilibrium : The force in strut AB can be obtained directly by summing moments about point C.

$$\circlearrowleft + \Sigma M_C = 0; \quad 2(1) - 6(1\tan 20°) + F_{AB}\sin 50°(0.4) - F_{AB}\cos 50°(0.4\tan 20°) = 0$$

$$F_{AB} = 0.8637 \text{ kN} = 0.864 \text{ kN} \qquad \textbf{Ans}$$

Using the result $F_{AB} = 0.8637$ kN and sum forces along x and y axes, we have,

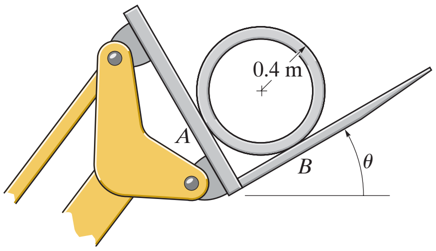
$$+\uparrow \Sigma F_y = 0; \quad 6 + 0.8637\cos 50° - C_y = 0$$

$$C_y = 6.56 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.8637\sin 50° + 2 - C_x = 0$$

$$C_x = 2.66 \text{ kN} \qquad \textbf{Ans}$$

***5–28.** The 1.4-Mg drainpipe is held in the tines of the fork lift. Determine the normal forces at A and B as functions of the blade angle θ and plot the results of force (vertical axis) versus θ (horizontal axis) for $0 \leq \theta \leq 90°$.

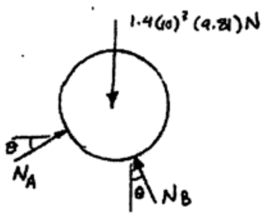


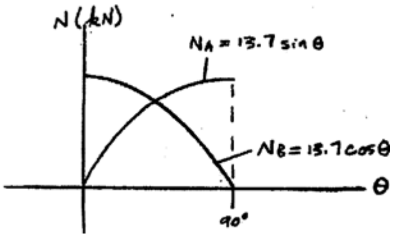
$+\nearrow \Sigma F_x = 0;\quad N_A - 1.4(10)^3(9.81)\sin\theta = 0$

$N_A = 13.7 \sin\theta$ kN **Ans**

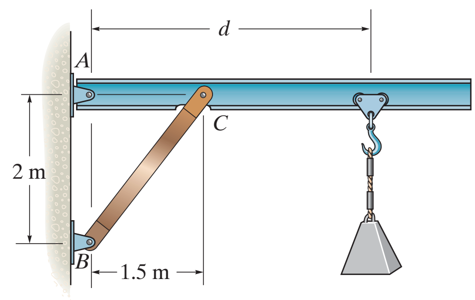
$+\nwarrow \Sigma F_y = 0;\quad N_B - 1.4(10)^3(9.81)\cos\theta = 0$

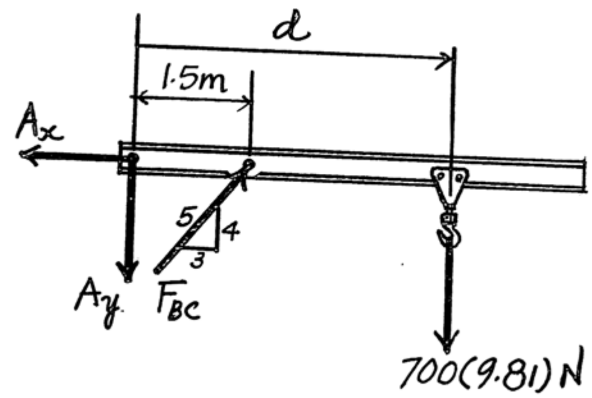
$N_B = 13.7 \cos\theta$ kN **Ans**





•5–29. The mass of 700 kg is suspended from a trolley which moves along the crane rail from $d = 1.7$ m to $d = 3.5$ m. Determine the force along the pin-connected knee strut BC (short link) and the magnitude of force at pin A as a function of position d. Plot these results of F_{BC} and F_A (vertical axis) versus d (horizontal axis).





$\circlearrowleft + \Sigma M_A = 0;\quad F_{BC}\left(\frac{4}{5}\right)(1.5) - 700(9.81)(d) = 0$

$F_{BC} = 5722.5d$ **Ans**

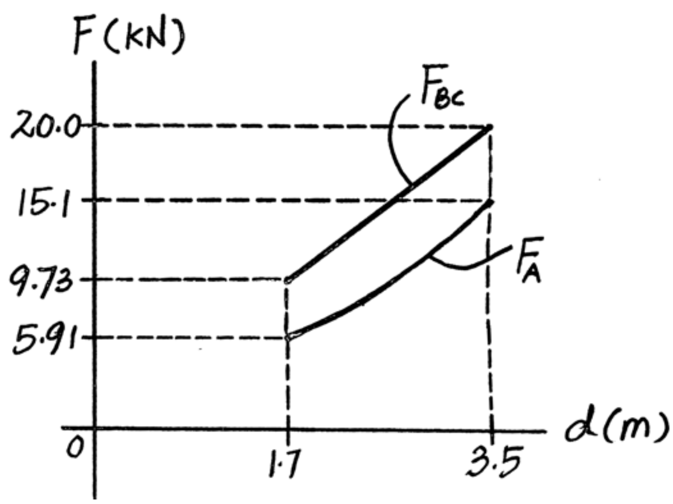
$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + (5722.5d)\left(\frac{3}{5}\right) = 0$

$A_x = 3433.5d$

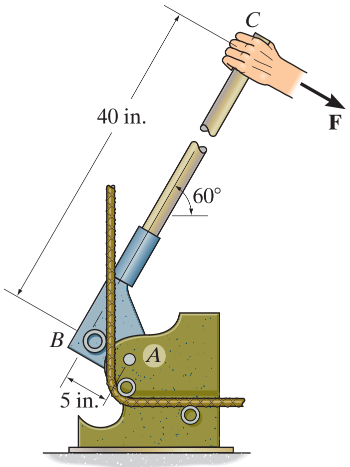
$+\uparrow \Sigma F_y = 0;\quad -A_y + (5722.5d)\left(\frac{4}{5}\right) - 700(9.81) = 0$

$A_y = 4578d - 6867$

$F_A = \sqrt{(3433.5d)^2 + (4578d - 6867)^2}$ **Ans**



5–30. If the force of $F = 100$ lb is applied to the handle of the bar bender, determine the horizontal and vertical components of reaction at pin A and the reaction of the roller B on the smooth bar.



Equations of Equilibrium: From the free - body diagram of the handle of the bar bender, Fig. a, A_y and N_B can be obtained by writing the force equation of equilibrium along the y axis and the moment equation of equilibrium about point A, respectively.

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 100\sin 30° = 0$

$A_y = 50$ lb **Ans.**

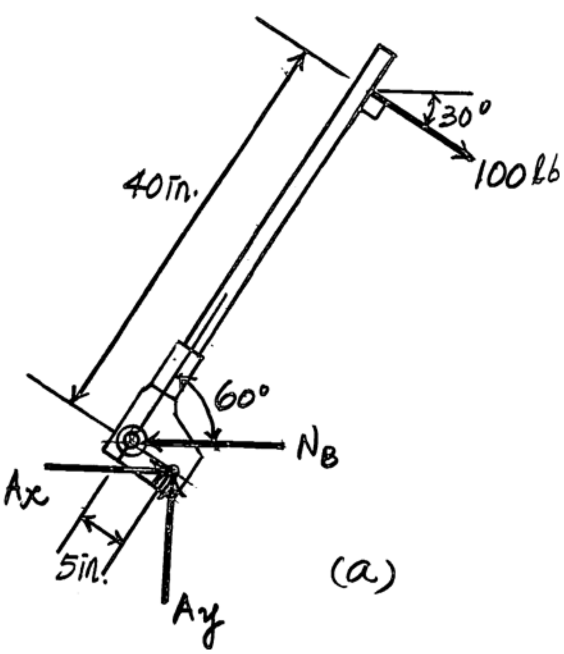
$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B \cos 60°(5) - 100(40) = 0$

$N_B = 1600\text{N} = 1.60$ kip **Ans.**

Using the result $N_B = 1600$ N and writing the force equation of equilibrium along the x axis,

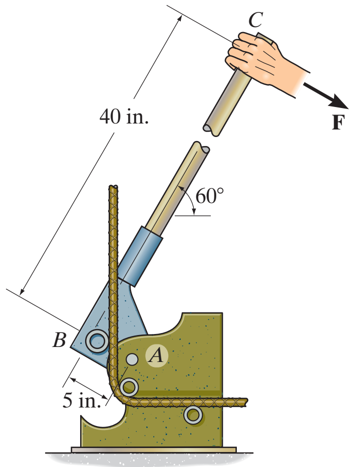
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 1600 + 100\cos 30° = 0$

$A_x = 1513.40 \text{ N} = 1.51$ kip **Ans.**



(a)

5–31. If the force of the smooth roller at B on the bar bender is required to be 1.5 kip, determine the horizontal and vertical components of reaction at pin A and the required magnitude of force $\mathbf{F}$ applied to the handle.



Equations of Equilibrium: From the free - body diagram of the handle of the bar bender, Fig. a, force F can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;\qquad 1500\cos 60°(5) - F(40) = 0$

$F = 93.75\text{ lb}$ **Ans.**

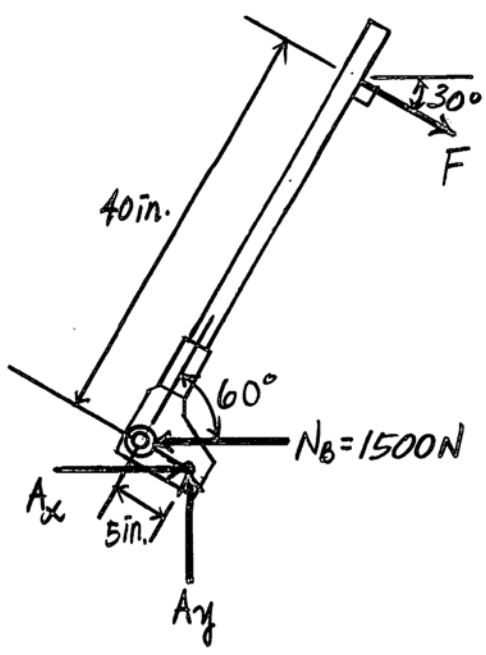
Using the above result and writing the force equation of equilibrium along the x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x + 93.75\cos 30° - 1500 = 0$

$A_x = 1418.81\text{ lb} = 1.42\text{ kip}$ **Ans.**

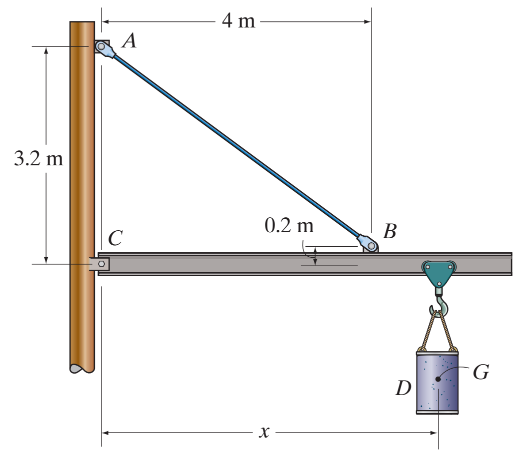
$+\uparrow \Sigma F_y = 0;\qquad A_y - 93.75\sin 30° = 0$

$A_y = 46.875\text{ lb} = 46.9\text{ lb}$ **Ans.**



(a)

***5–32.** The jib crane is supported by a pin at C and rod AB. If the load has a mass of 2 Mg with its center of mass located at G, determine the horizontal and vertical components of reaction at the pin C and the force developed in rod AB on the crane when $x = 5$ m.



Equations of Equilibrium: Realizing that rod AB is a two - force member, it will exert a force $\mathbf{F}_{AB}$ directed along its axis on the beam, as shown on the free - body diagram in Fig. *a*. From the free - body diagram, F_{AB} can be obtained by writing the moment equation of equilibrium about point C.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}\left(\frac{3}{5}\right)(4) + F_{AB}\left(\frac{4}{5}\right)(0.2) - 2000(9.81)(5) = 0$$

$$F_{AB} = 38\,320.31 \text{ N} = 38.3 \text{ kN} \qquad \textbf{Ans.}$$

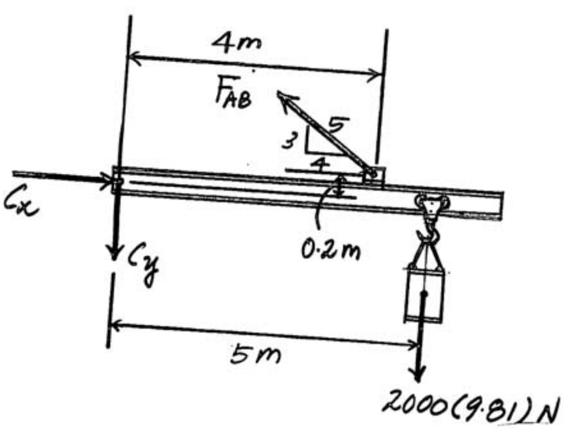
Using the above result and writing the force equations of equilibrium along the x and y axes.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x - 38\,320.31\left(\frac{4}{5}\right) = 0$$

$$C_x = 30\,656.25 \text{ N} = 30.7 \text{ kN} \qquad \textbf{Ans.}$$

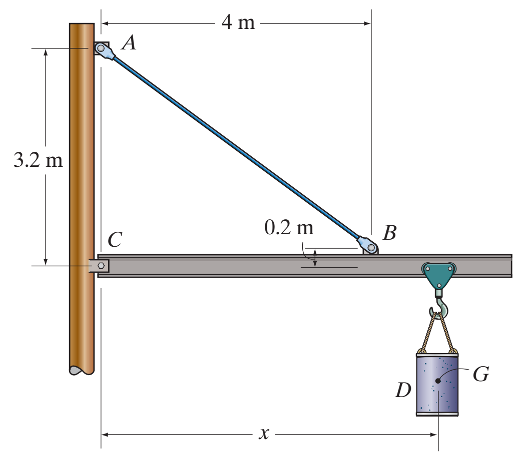
$$+\uparrow \Sigma F_y = 0; \qquad 38\,320.31\left(\frac{3}{5}\right) - 2000(9.81) - C_y = 0$$

$$C_y = 3372.19 \text{ N} = 3.37 \text{ kN} \qquad \textbf{Ans.}$$



(a)

•5–33. The jib crane is supported by a pin at C and rod AB. The rod can withstand a maximum tension of 40 kN. If the load has a mass of 2 Mg, with its center of mass located at G, determine its maximum allowable distance x and the corresponding horizontal and vertical components of reaction at C.



Equations of Equilibrium: Realizing that rod AB is a two - force member, it will exert a force $\mathbf{F}_{AB}$ directed along its axis on the beam, as shown on the free - body diagram in Fig. *a*. From the free - body diagram, the distance x can be obtained by writing the moment equation of equilibrium about point C.

$$\curvearrowleft +\Sigma M_C = 0; \qquad 40\,000\left(\frac{3}{5}\right)(4) + 40\,000\left(\frac{4}{5}\right)(0.2) - 2000(9.81)(x) = 0$$

$$x = 5.22 \text{ m} \qquad \textbf{Ans.}$$

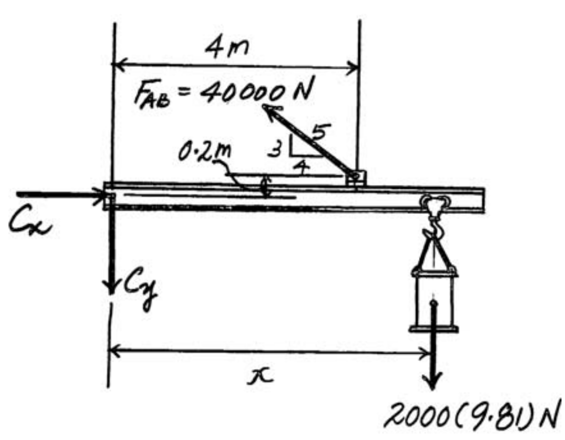
Writing the force equations of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad C_x - 40\,000\left(\frac{4}{5}\right) = 0$$

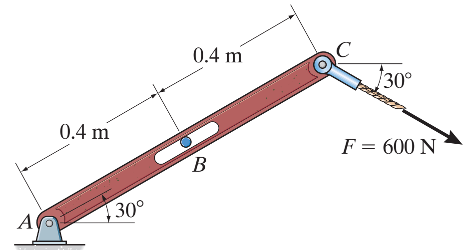
$$C_x = 32\,000 \text{ N} = 32 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 40\,000\left(\frac{3}{5}\right) - 2000(9.81) - C_y = 0$$

$$C_y = 4380 \text{ N} = 4.38 \text{ kN} \qquad \textbf{Ans.}$$



5–34. Determine the horizontal and vertical components of reaction at the pin A and the normal force at the smooth peg B on the member.



Equations of Equilibrium: From the free - body diagram of the member, Fig. *a*, N_B can be obtained by writing the moment equation of equilibrium about point A.

$$\circlearrowleft +\Sigma M_A = 0; \qquad N_B(0.4) - 600\cos 30°(0.8) = 0$$

$$N_B = 1039.23\text{ N} = 1.04\text{ kN} \qquad \textbf{Ans.}$$

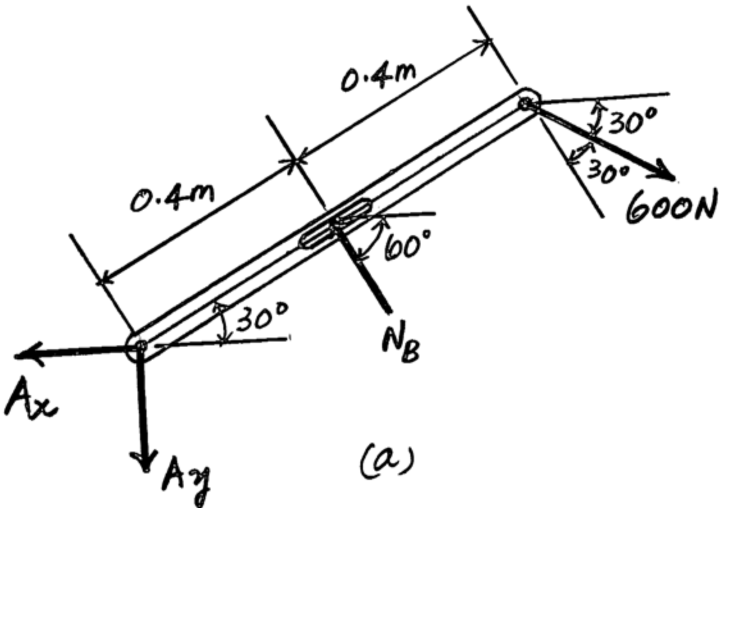
Using this result and writing the force equations of equilibrium along the x and y axes,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600\cos 30° - 1039.23\cos 60° - A_x = 0$$

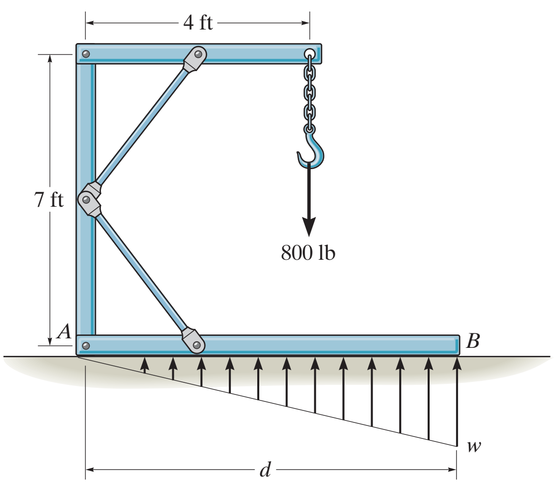
$$A_x = 0 \qquad \textbf{Ans.}$$

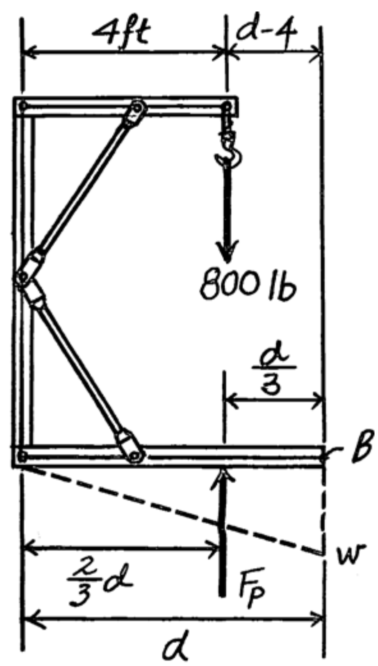
$$+\uparrow \Sigma F_y = 0; \qquad -A_y + 1039.23\sin 60° - 600\sin 30° = 0$$

$$A_y = 600\text{ N} \qquad \textbf{Ans.}$$



5–35. The framework is supported by the member AB which rests on the smooth floor. When loaded, the pressure distribution on AB is linear as shown. Determine the length d of member AB and the intensity w for this case.





$+\uparrow \Sigma F_y = 0;\quad F_P - 800 = 0$

$F_P = 800 \text{ lb}$

When tipping;

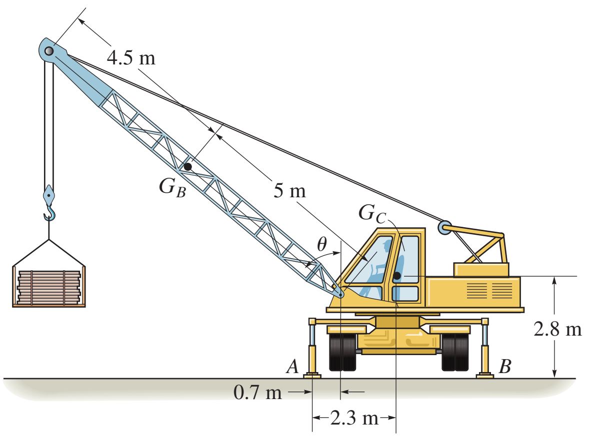
$\circlearrowleft + \Sigma M_B = 0;\quad -800\left(\frac{d}{3}\right) + 800(d-4) = 0$

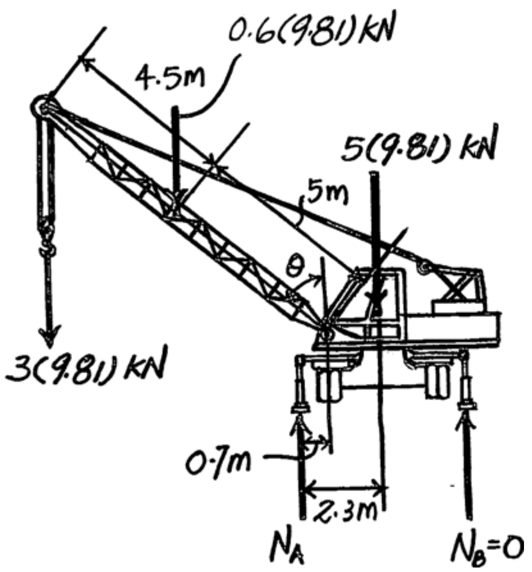
$d = 6 \text{ ft}$ **Ans**

$F_P = \frac{1}{2}wd = \frac{1}{2}(w)(6) = 800$

$w = 267 \text{ lb/ft}$ **Ans**

***5–36.** Outriggers *A* and *B* are used to stabilize the crane from overturning when lifting large loads. If the load to be lifted is 3 Mg, determine the *maximum* boom angle θ so that the crane does not overturn. The crane has a mass of 5 Mg and center of mass at G_C, whereas the boom has a mass of 0.6 Mg and center of mass at G_B.



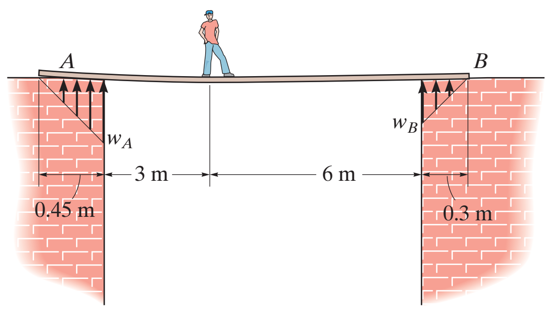


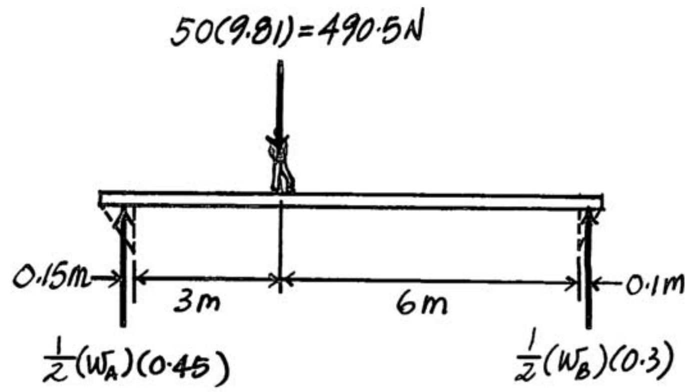
$\curvearrowleft + \Sigma M_A = 0;\quad -5(9.81)(2.3) + 3(9.81)(9.5\sin\theta - 0.7)$

$+ 0.6(9.81)(5\sin\theta - 0.7) = 0$

$\theta = 26.4°$ **Ans**

•5–37. The wooden plank resting between the buildings deflects slightly when it supports the 50-kg boy. This deflection causes a triangular distribution of load at its ends, having maximum intensities of w_A and w_B. Determine w_A and w_B, each measured in N/m, when the boy is standing 3 m from one end as shown. Neglect the mass of the plank.





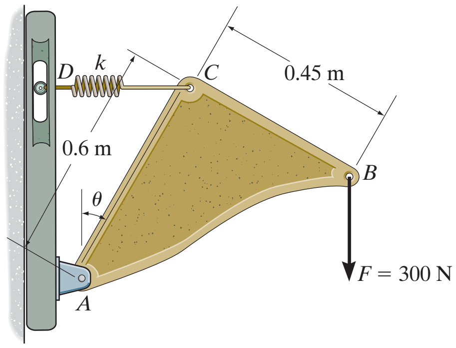
$\curvearrowleft + \Sigma M_A = 0; \quad -490.5\,(3.15) + \frac{1}{2} w_B\,(0.3)\,(9.25) = 0$

$w_B = 1113.6 \text{ N/m} = 1.11 \text{ kN/m}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2} w_A\,(0.45) + \frac{1}{2}\,(1113.6)\,(0.3) - 490.5 = 0$

$w_A = 1437.6 \text{ N/m} = 1.44 \text{ kN/m}$ **Ans**

5–38. Spring CD remains in the horizontal position at all times due to the roller at D. If the spring is unstretched when $\theta = 0°$ and the bracket achieves its equilibrium position when $\theta = 30°$, determine the stiffness k of the spring and the horizontal and vertical components of reaction at pin A.



Spring Force Formula: At the equilibrium position, the spring elongates $x = 0.6 \sin 30°$ m. Using the spring force formula, the force in spring CD is found to be $F_{sp} = kx = 0.3k$.

Equations of Equilibrium: From the free - body diagram of the bracket, Fig. a, the stiffness k of spring CD and A_y can be obtained by writing the moment equation of equilibrium about point A and the force equation of equilibrium along the x axis, respectively.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 0.3k \cos 30°(0.6) - 300 \cos 30°(0.45) - 300 \sin 30°(0.6) = 0$

$k = 1327.35 \text{ N/m} = 1.33 \text{ kN/m}$ **Ans.**

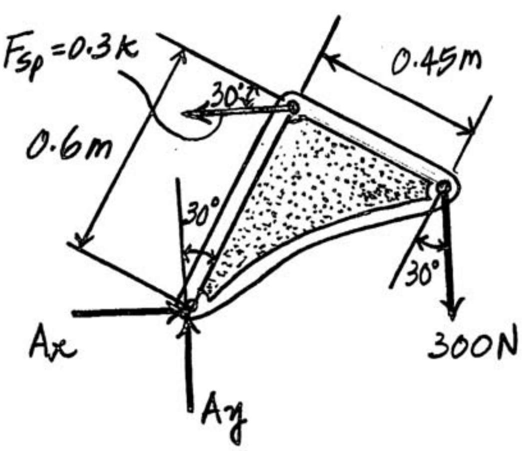
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 300 = 0$

$A_y = 300 \text{ N}$ **Ans.**

Using the result $k = 1327.35$ N / m and writing the force equation of equilibrium along the x axis,

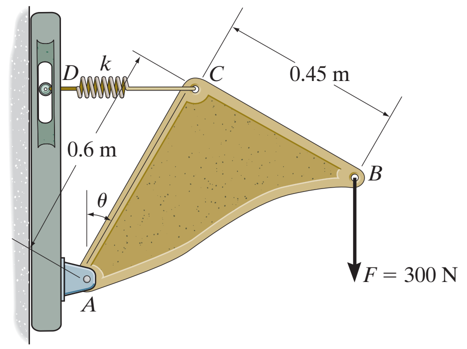
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 0.3(1327.35) = 0$

$A_x = 398.21 \text{ N} = 398 \text{ N}$ **Ans.**



(a)

5–39. Spring *CD* remains in the horizontal position at all times due to the roller at *D*. If the spring is unstretched when $\theta = 0°$ and the stiffness is $k = 1.5$ kN/m, determine the smallest angle θ for equilibrium and the horizontal and vertical components of reaction at pin *A*.



Spring Force Formula: At the equilibrium position, the spring elongates $x = 0.6\sin\theta$. Using the spring force formula, the force in spring *CD* is found to be $F_{sp} = kx = 1500(0.6\sin\theta) = 900\sin\theta$.

Equations of Equilibrium: From the free - body diagram of the bracket, Fig. *a*, the equilibrium position θ and A_y can be obtained by writing the moment equation of equilibrium about point *A* and the force equation of equilibrium along the *y* axis, respectively.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 900\sin\theta\cos\theta(0.6) - 300\sin\theta(0.6) - 300\cos\theta(0.45) = 0$

$$540\sin\theta\cos\theta - 180\sin\theta - 135\cos\theta = 0$$

Solving by trial and error yields

$$\theta = 23.083° = 23.1° \qquad \textbf{Ans.}$$

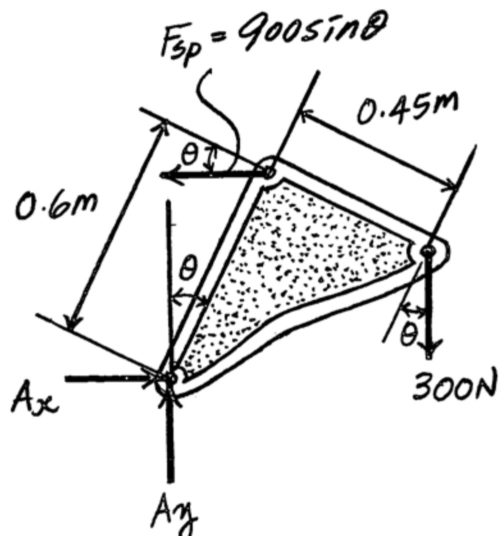
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 300 = 0$

$$A_y = 300\text{ N} \qquad \textbf{Ans.}$$

Using the result $\theta = 23.083°$ and writing the force equation of equilibrium along the *x* axis,

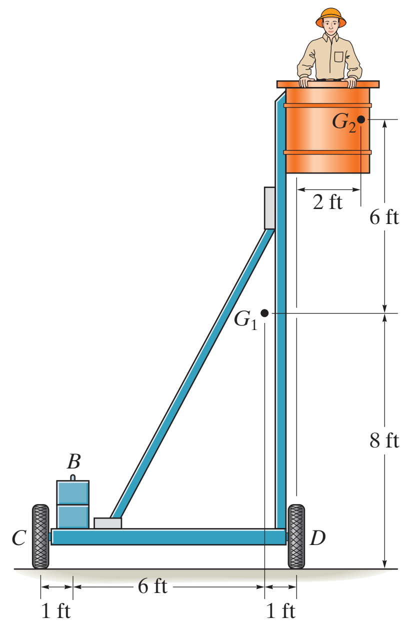
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 900\sin 23.083° = 0$

$$A_x = 352.86\text{ N} = 353\text{ N} \qquad \textbf{Ans.}$$



(a)

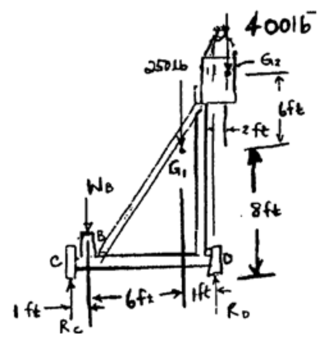
***5–40.** The platform assembly has a weight of 250 lb and center of gravity at G_1. If it is intended to support a maximum load of 400 lb placed at point G_2, determine the smallest counterweight W that should be placed at B in order to prevent the platform from tipping over.



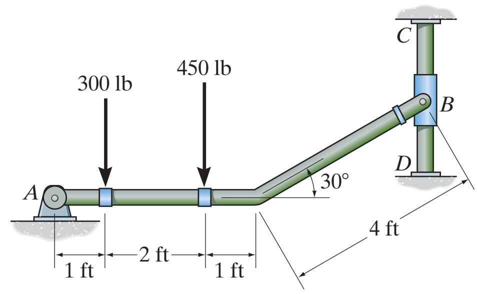
When tipping occurs, $R_C = 0$

$\circlearrowleft + \Sigma M_D = 0;\qquad -400(2) + 250(1) + W_B(7) = 0$

$W_B = 78.6 \text{ lb}$ **Ans**



•5–41. Determine the horizontal and vertical components of reaction at the pin *A* and the reaction of the smooth collar *B* on the rod.



Equations of Equilibrium: From the free - body diagram, A_y and N_B can be obtained by writing the force equation of equilibrium along the y axis and the moment equation of equilibrium about point A.

$+\uparrow \Sigma F_y = 0;\quad A_y - 300 - 450 = 0$

$A_y = 750\text{ lb}$ **Ans.**

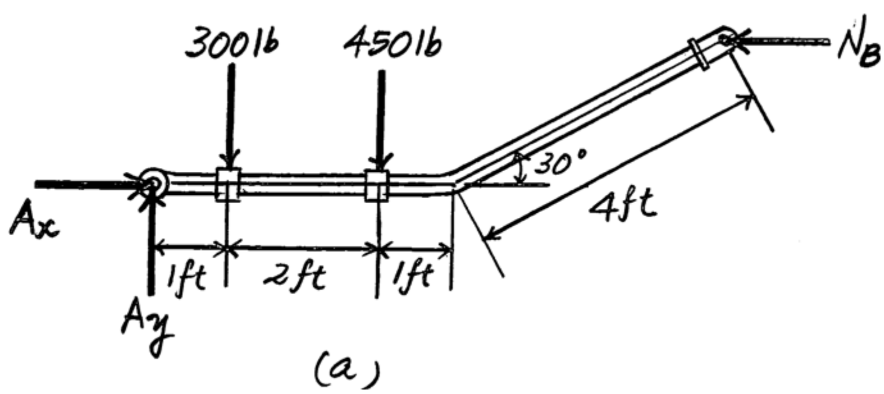
$\circlearrowleft + \Sigma M_A = 0;\quad N_B(4\sin 30°) - 300(1) - 450(3) = 0$

$N_B = 825\text{ lb}$ **Ans.**

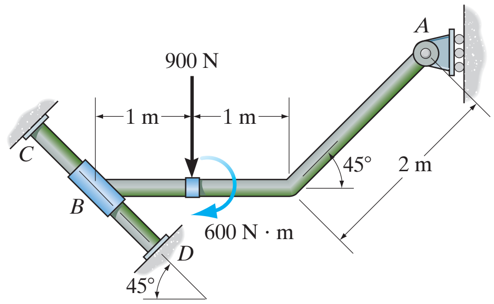
Using the result $N_B = 825\text{ lb}$ and writing the force equation of equilibrium along the x axis,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad A_x - 825 = 0$

$A_x = 825\text{ lb}$ **Ans.**



5–42. Determine the support reactions of roller A and the smooth collar B on the rod. The collar is fixed to the rod AB, but is allowed to slide along rod CD.



Equations of Equilibrium: From the free - body diagram of the rod, Fig. *a*, N_B can be obtained by writing the force equation of equilibrium along the *y* axis.

$+\uparrow \Sigma F_y = 0;\quad N_B \sin 45° - 900 = 0$

$N_B = 1272.79\text{ N} = 1.27\text{ kN}$ **Ans.**

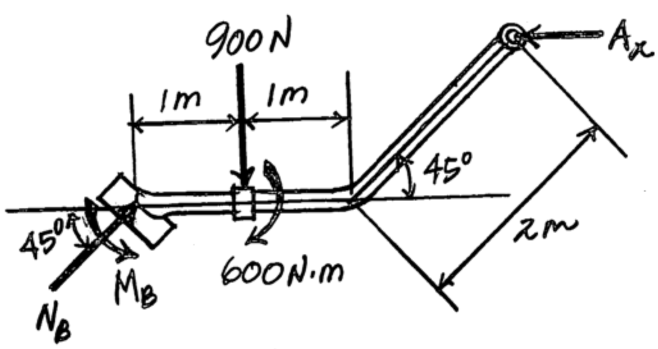
Using the above result and writing the force equation of equilibrium and the moment equation of equilibrium about point B,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 1272.79 \cos 45° - A_x = 0$

$A_x = 900\text{ N}$ **Ans.**

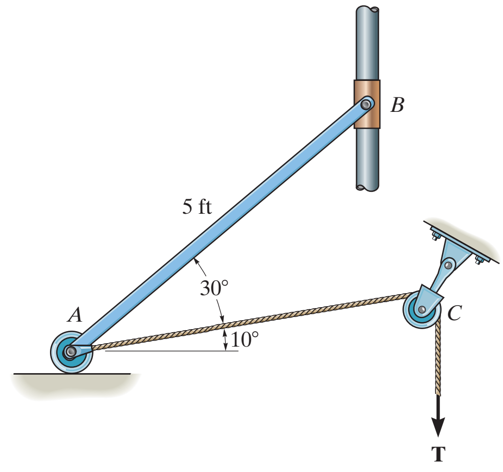
$\curvearrowleft + \Sigma M_B = 0;\quad -900(1) + 900(2)\sin 45° - 600 + M_B = 0$

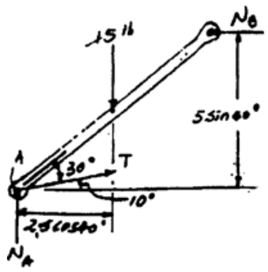
$M_B = 227\text{ N}\cdot\text{m}$ **Ans.**



(a)

5–43. The uniform rod *AB* has a weight of 15 lb. Determine the force in the cable when the rod is in the position shown.

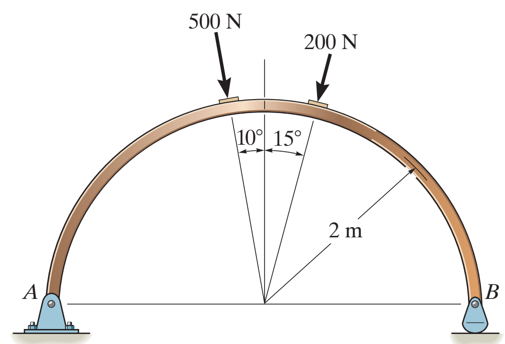


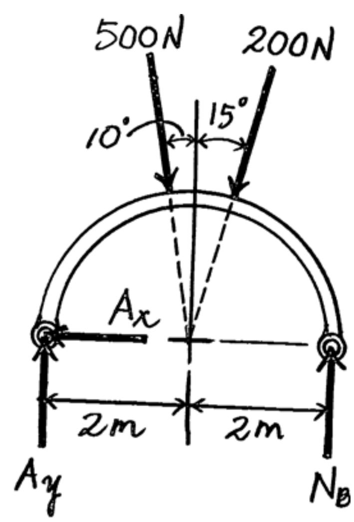


$\curvearrowleft + \Sigma M_A = 0;\quad N_B(5\sin 40°) - 15(2.5\cos 40°) = 0 \quad N_B = 8.938\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad T\cos 10° - 8.938 = 0 \quad T = 9.08\text{ lb}$ **Ans**

***5–44.** Determine the horizontal and vertical components of force at the pin *A* and the reaction at the rocker *B* of the curved beam.





$\curvearrowleft + \Sigma M_A = 0;\quad N_B(4) - 200\cos 15°(2) - 500\cos 10°(2) = 0$

$N_B = 342.79 = 343\text{ N}$ **Ans**

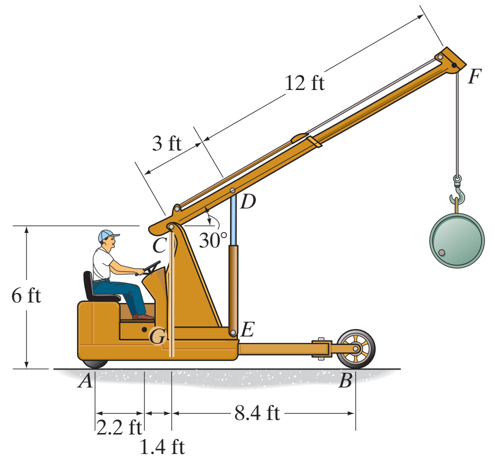
$+\uparrow \Sigma F_y = 0;\quad A_y - 500\cos 10° - 200\cos 15° + 342.79 = 0$

$A_y = 342.8 = 343\text{ N}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad -A_x + 500\sin 10° - 200\sin 15° = 0$

$A_x = 35.1\text{ N}$ **Ans**

•5–45. The floor crane and the driver have a total weight of 2500 lb with a center of gravity at *G*. If the crane is required to lift the 500-lb drum, determine the normal reaction on *both* the wheels at *A* and *both* the wheels at *B* when the boom is in the position shown.



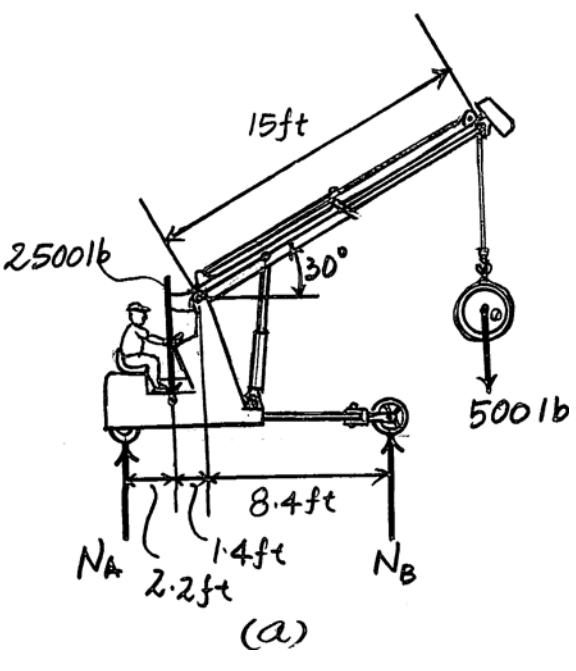
Equations of Equilibrium: From the free - body diagram of the floor crane, Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 2500(1.4 + 8.4) - 500(15\cos 30° - 8.4) - N_A(2.2 + 1.4 + 8.4) = 0$

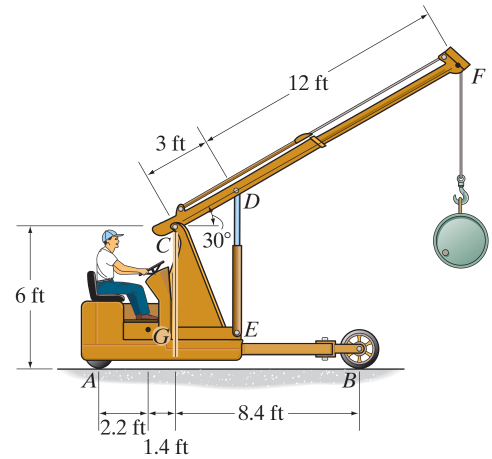
$N_A = 1850.40 \text{ lb} = 1.85 \text{ kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 1850.40 - 2500 - 500 + N_B = 0$

$N_B = 1149.60 \text{ lb} = 1.15 \text{ kip}$ **Ans.**



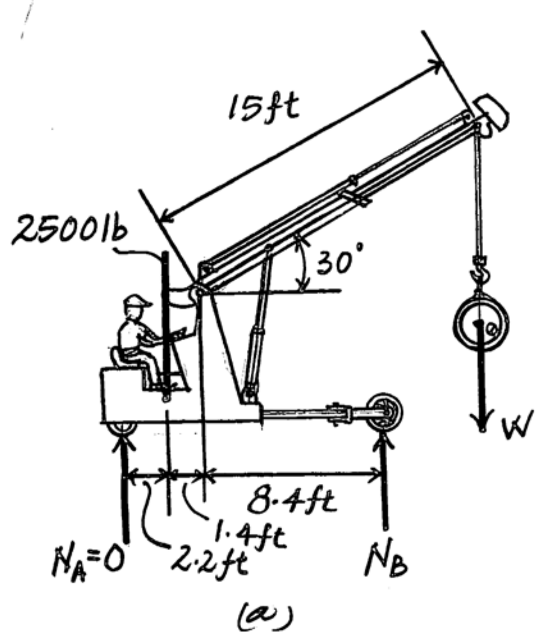
5–46. The floor crane and the driver have a total weight of 2500 lb with a center of gravity at G. Determine the largest weight of the drum that can be lifted without causing the crane to overturn when its boom is in the position shown.



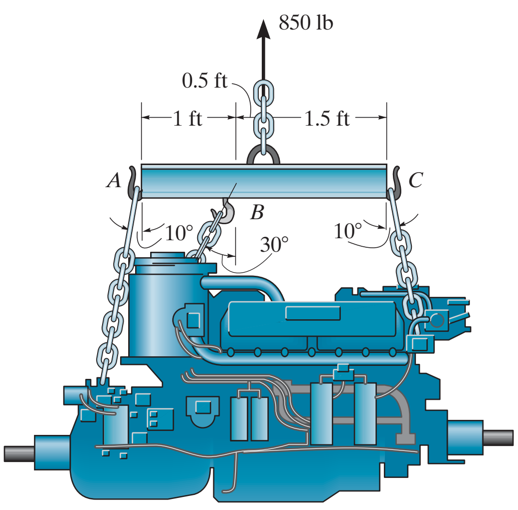
Equations of Equilibrium: Since the floor crane tends to overturn about point B, the wheel at A will leave the ground and $N_A = 0$. From the free - body diagram of the floor crane, Fig. a, W can be obtained by writing the moment equation of equilibrium about point B.

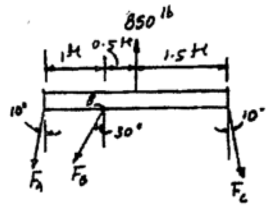
$\circlearrowleft + \Sigma M_B = 0;$ $\quad 2500(1.4 + 8.4) - W(15\cos 30° - 8.4) = 0$

$W = 5337.25 \text{ lb} = 5.34 \text{ kip}$ **Ans.**



5–47. The motor has a weight of 850 lb. Determine the force that each of the chains exerts on the supporting hooks at A, B, and C. Neglect the size of the hooks and the thickness of the beam.





$\curvearrowleft + \Sigma M_B = 0;\quad F_A \cos 10°(1) + 850(0.5) - F_C \cos 10°(2) = 0 \qquad (1)$

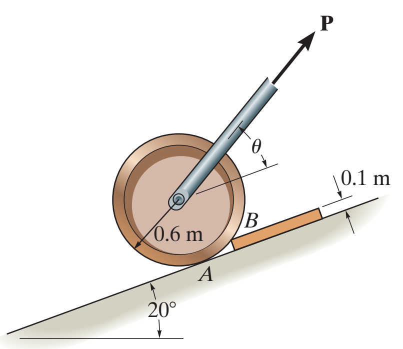
$\xrightarrow{+} \Sigma F_x = 0;\quad F_C \sin 10° - F_B \sin 30° - F_A \sin 10° = 0 \qquad (2)$

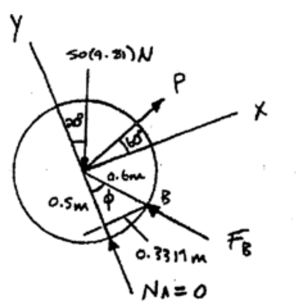
$+\uparrow \Sigma F_y = 0;\quad 850 - F_A \cos 10° - F_B \cos 30° - F_C \cos 10° = 0 \qquad (3)$

Solving Eqs. (1), (2) and (3) yields :

$F_A = 432$ lb $\quad F_B = 0 \quad F_C = 432$ lb **Ans**

***5–48.** Determine the force P needed to pull the 50-kg roller over the smooth step. Take $\theta = 60°$.





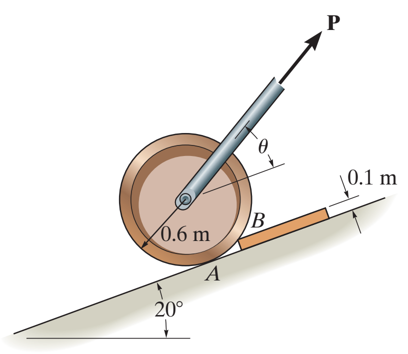
$\phi = \cos^{-1}\left(\frac{0.5}{0.6}\right) = 33.56°$

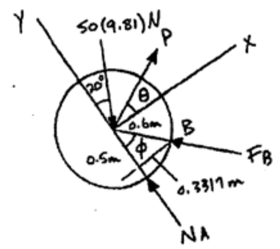
$\curvearrowleft + \Sigma M_B = 0;\quad 50\,(9.81) \sin 20° \,(0.5) + 50(9.81) \cos 20° (0.3317) - P \cos 60° \,(0.5)$

$- P \sin 60° \,(0.3317) = 0$

$P = 441$ N. **Ans**

•5–49. Determine the magnitude and direction θ of the minimum force P needed to pull the 50-kg roller over the smooth step.





For P_{min}, $N_A \to 0$, $\phi = \cos^{-1}\left(\frac{0.5}{0.6}\right) = 33.56°$

$$\circlearrowleft + \Sigma M_B = 0; \quad 50\,(9.81)\sin 20°\,(0.5) + 50(9.81)\cos 20°\,(0.3317) - P\cos\theta\,(0.5)$$

$$- P\sin\theta\,(0.3317) = 0$$

$$236.75 - P\cos\theta\,(0.5) - P\sin\theta\,(0.3317) = 0$$

$$P = \frac{236.75}{(0.5\cos\theta + 0.3317\sin\theta)}$$

For P_{min};

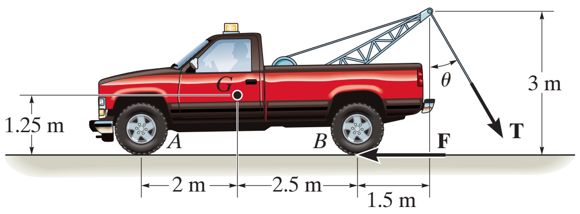
$$\frac{dP}{d\theta} = \frac{-236.75\,(-0.5\sin\theta + 0.3317\cos\theta)}{(0.5\cos\theta + 0.3317\sin\theta)^2} = 0$$

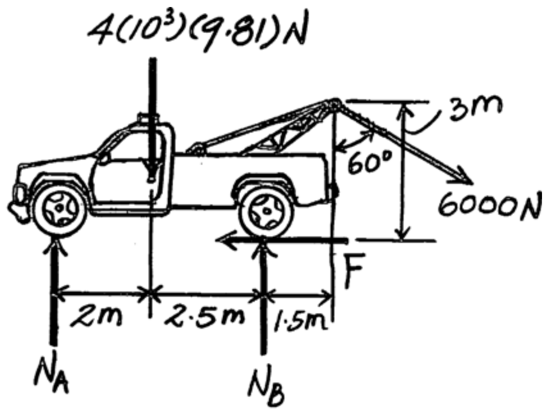
$$\tan\theta = \frac{0.3317}{0.5}$$

$\theta = 33.6°$ **Ans**

$P_{min} = 395\text{ N}$ **Ans**

5–50. The winch cable on a tow truck is subjected to a force of $T = 6$ kN when the cable is directed at $\theta = 60°$. Determine the magnitudes of the total brake frictional force **F** for the rear set of wheels B and the total normal forces at *both* front wheels A and both rear wheels B for equilibrium. The truck has a total mass of 4 Mg and mass center at G.





$\xrightarrow{+} \Sigma F_x = 0; \quad 6000 \sin 60° - F = 0$

$F = 5196 \text{ N} = 5.20 \text{ kN}$ **Ans**

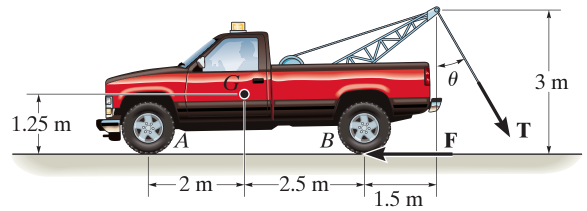
$\circlearrowleft + \Sigma M_B = 0; \quad -N_A (4.5) + 4(10^3)(9.81)(2.5) - 6000 \sin 60° (3) - 6000 \cos 60° (1.5) = 0$

$N_A = 17\,336 \text{ N} = 17.3 \text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 17\,336 - 4(10^3)(9.81) - 6000 \cos 60° + N_B = 0$

$N_B = 24\,904 \text{ N} = 24.9 \text{ kN}$ **Ans**

5–51. Determine the minimum cable force T and critical angle θ which will cause the tow truck to start tipping, i.e., for the normal reaction at A to be zero. Assume that the truck is braked and will not slip at B. The truck has a total mass of 4 Mg and mass center at G.x



$$\circlearrowleft + \Sigma M_B = 0; \quad 4(10^3)(9.81)(2.5) - T\sin\theta\,(3) - T\cos\theta\,(1.5) = 0$$

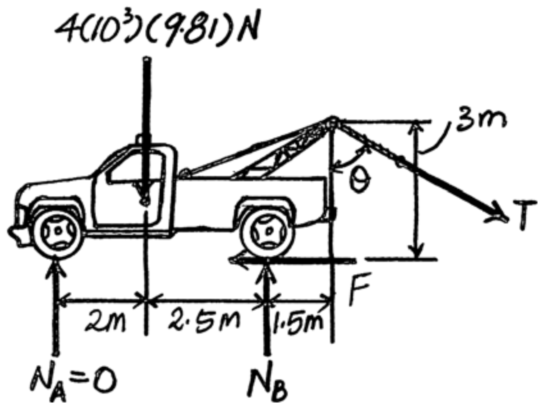
$$T = \frac{65\,400}{(\cos\theta + 2\sin\theta)}$$

$$\frac{dT}{d\theta} = \frac{-65\,400(-\sin\theta + 2\cos\theta)}{(\cos\theta + 2\sin\theta)^2} = 0$$

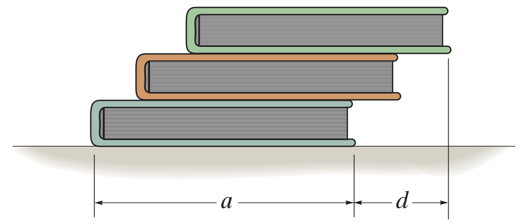
$$-\sin\theta + 2\cos\theta = 0$$

$$\theta = \tan^{-1}2 = 63.43° = 63.4° \qquad \textbf{Ans}$$

$$T = \frac{65\,400}{(\cos 63.43° + 2\sin 63.43°)} = 29.2\text{ kN} \qquad \textbf{Ans}$$



***5–52.** Three uniform books, each having a weight W and length a, are stacked as shown. Determine the maximum distance d that the top book can extend out from the bottom one so the stack does not topple over.

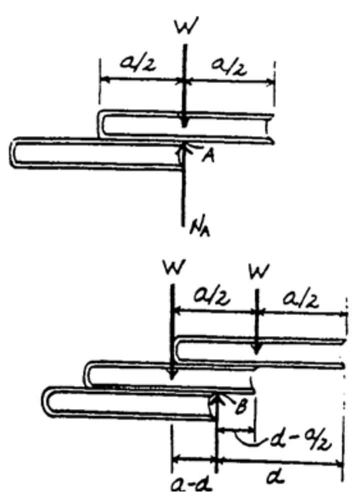


Equilibrium : For top two books, the upper book will topple when the center of gravity of this book is to the right of point A. Therefore, the maximum distance from the right edge of this book to point A is $a/2$.

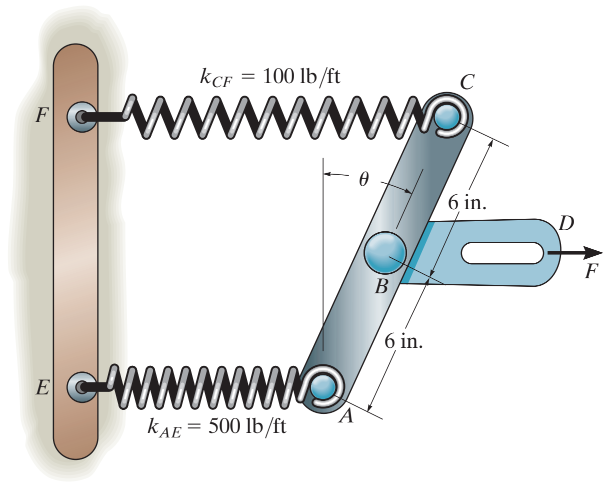
Equation of Equilibrium : For the entire three books, the top two books will topple about point B.

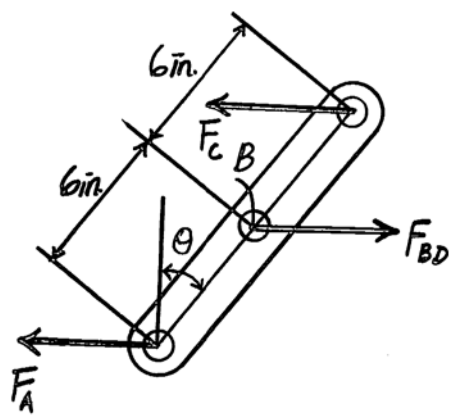
$$\circlearrowleft + \Sigma M_B = 0; \quad W(a-d) - W\left(d - \frac{a}{2}\right) = 0$$

$$d = \frac{3a}{4} \qquad \textbf{Ans}$$



•5–53. Determine the angle θ at which the link ABC is held in equilibrium if member BD moves 2 in. to the right. The springs are originally unstretched when $\theta = 0°$. Each spring has the stiffness shown. The springs remain horizontal since they are attached to roller guides.





$\circlearrowleft + \Sigma M_B = 0; \quad F_C(6\cos\theta) - F_A(6\cos\theta) = 0 \qquad F_C = F_A = F$

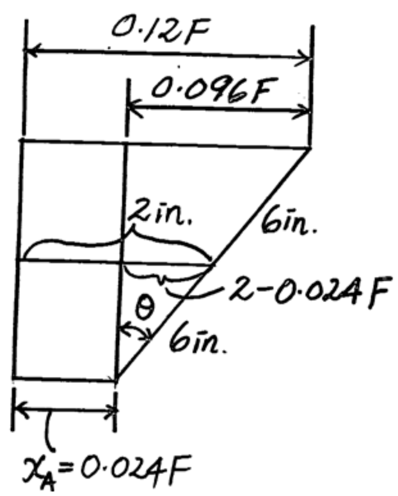
$x_C = \dfrac{F}{\left(\frac{100}{12}\right)} = 0.12F$ and $x_A = \dfrac{F}{\left(\frac{500}{12}\right)} = 0.024F$

Using similar triangles

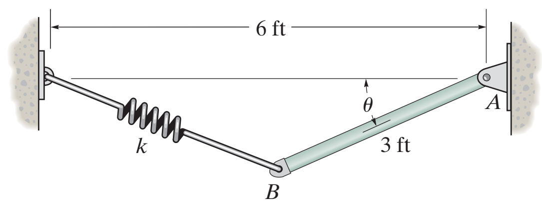
$\dfrac{0.096F}{12} = \dfrac{2 - 0.024F}{6} \qquad F = 27.77$ lb

$\sin\theta = \dfrac{0.096(27.77)}{12} = 0.2222$

$\theta = 12.8°$ **Ans**



5–54. The uniform rod AB has a weight of 15 lb and the spring is unstretched when $\theta = 0°$. If $\theta = 30°$, determine the stiffness k of the spring.



Geometry : From triangle CDB, the cosine law gives

$$l = \sqrt{2.536^2 + 1.732^2 - 2(2.536)(1.732)\cos 120°} = 3.718 \text{ ft}$$

Using the sine law,

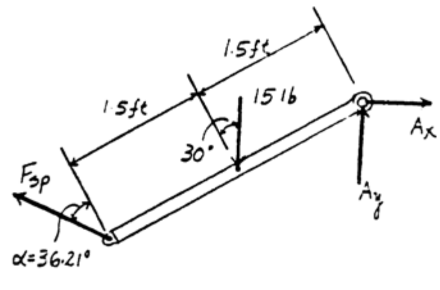
$$\frac{\sin\alpha}{2.536} = \frac{\sin 120°}{3.718} \qquad \alpha = 36.21°$$

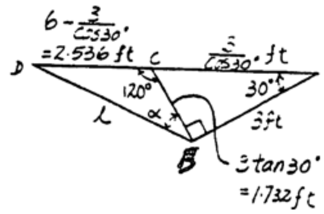
Equations of Equilibrium : The force in the spring can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad 15\cos 30°(1.5) - F_{sp}\cos 36.21°(3) = 0$$
$$F_{sp} = 8.050 \text{ lb}$$

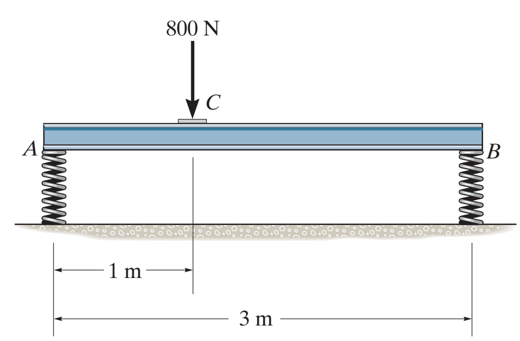
Spring Force Formula : The spring stretches $x = 3.718 - 3 = 0.718$ ft.

$$k = \frac{F_{sp}}{x} = \frac{8.050}{0.718} = 11.2 \text{ lb/ft} \qquad \textbf{Ans}$$





5–55. The horizontal beam is supported by springs at its ends. Each spring has a stiffness of $k = 5$ kN/m and is originally unstretched so that the beam is in the horizontal position. Determine the angle of tilt of the beam if a load of 800 N is applied at point C as shown.



Equations of Equilibrium : The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 800(2) - F_A(3) = 0 \qquad F_A = 533.33 \text{ N}$$

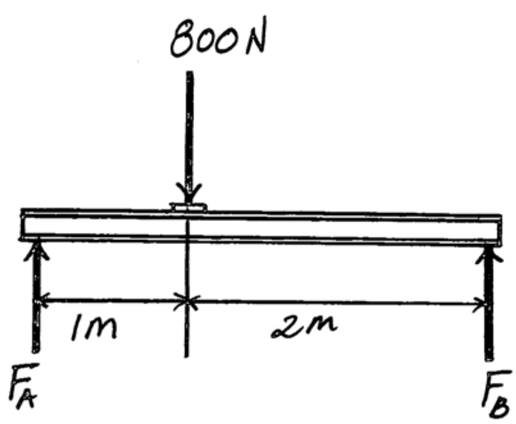
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(3) - 800(1) = 0 \qquad F_B = 266.67 \text{ N}$$

Spring Formula : Applying $\Delta = \frac{F}{k}$, we have

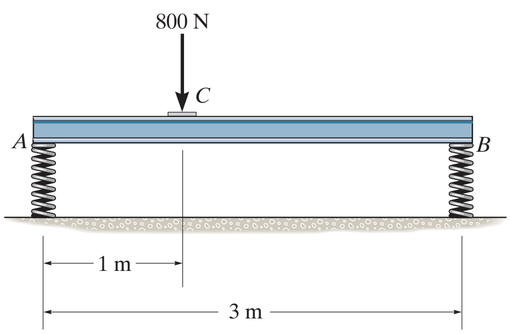
$$\Delta_A = \frac{533.33}{5(10^3)} = 0.1067 \text{ m}$$
$$\Delta_B = \frac{266.67}{5(10^3)} = 0.05333 \text{ m}$$

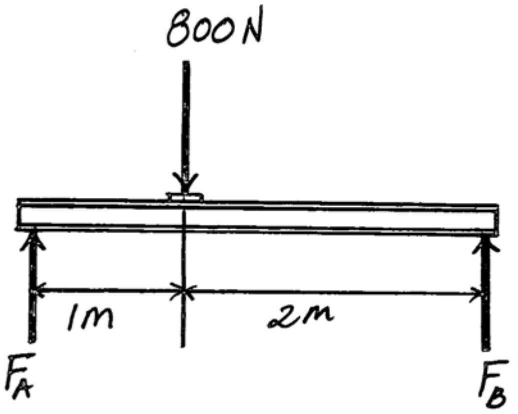
Geometry : The angle of tilt α is

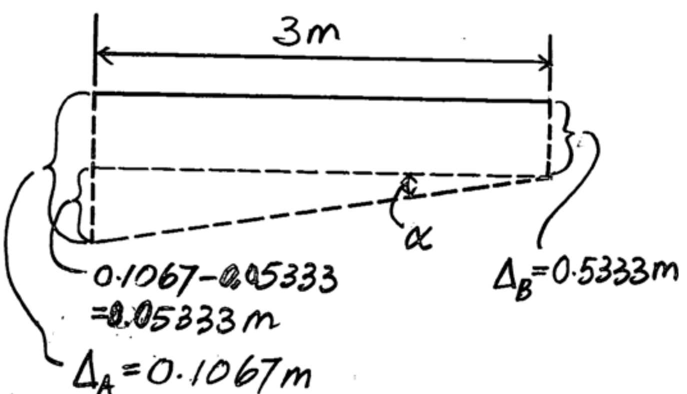
$$\alpha = \tan^{-1}\left(\frac{0.05333}{3}\right) = 1.02° \qquad \textbf{Ans}$$



***5–56.** The horizontal beam is supported by springs at its ends. If the stiffness of the spring at A is $k_A = 5\ \text{kN/m}$, determine the required stiffness of the spring at B so that if the beam is loaded with the 800 N it remains in the horizontal position. The springs are originally constructed so that the beam is in the horizontal position when it is unloaded.







Equations of Equilibrium : The spring forces at A and B can be obtained directly by summing moments about points B and A respectively.

$$\circlearrowleft + \Sigma M_B = 0; \quad 800(2) - F_A(3) = 0 \quad F_A = 533.33\ \text{N}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad F_B(3) - 800(1) = 0 \quad F_B = 266.67\ \text{N}$$

Spring Formula : Applying $\Delta = \dfrac{F}{k}$, we have

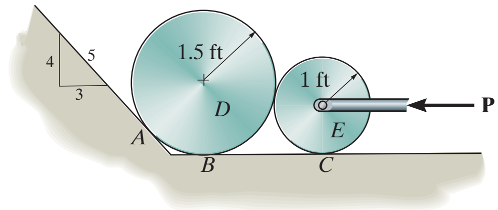
$$\Delta_A = \frac{533.33}{5(10^3)} = 0.1067\ \text{m} \qquad \Delta_B = \frac{266.67}{k_B}$$

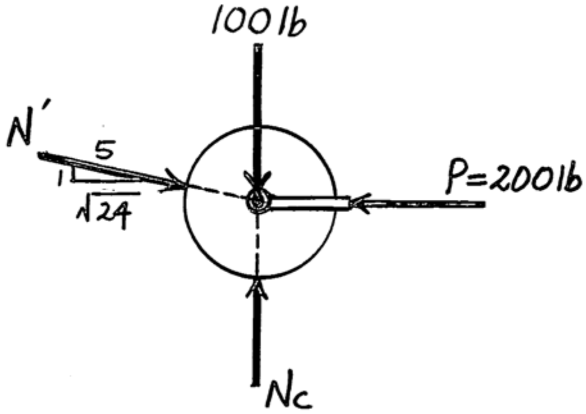
Geometry : Requires, $\Delta_B = \Delta_A$. Then

$$\frac{266.67}{k_B} = 0.1067$$

$$k_B = 2500\ \text{N/m} = 2.50\ \text{kN/m} \qquad \textbf{Ans}$$

•**5–57.** The smooth disks *D* and *E* have a weight of 200 lb and 100 lb, respectively. If a horizontal force of $P = 200$ lb is applied to the center of disk *E*, determine the normal reactions at the points of contact with the ground at *A*, *B*, and *C*.





For disk *E*:

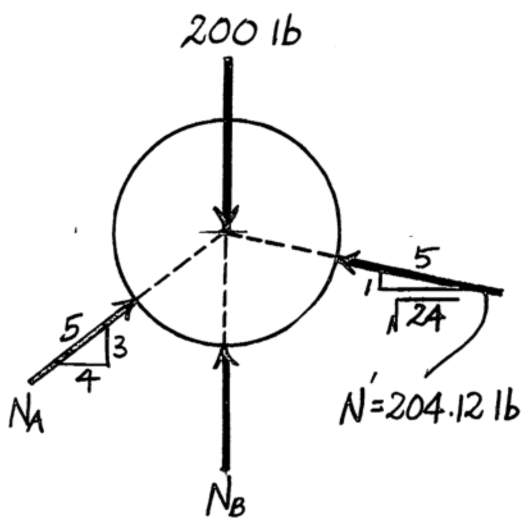
$\xrightarrow{+} \Sigma F_x = 0; \quad -P + N'\left(\frac{\sqrt{24}}{5}\right) = 0$

$+\uparrow \Sigma F_y = 0; \quad N_C - 100 - N'\left(\frac{1}{5}\right) = 0$

For disk *D*:

$\xrightarrow{+} \Sigma F_x = 0; \quad N_A\left(\frac{4}{5}\right) - N'\left(\frac{\sqrt{24}}{5}\right) = 0$

$+\uparrow \Sigma F_y = 0; \quad N_A\left(\frac{3}{5}\right) + N_B - 200 + N'\left(\frac{1}{5}\right) = 0$



Set $P = 200$ lb and solve:

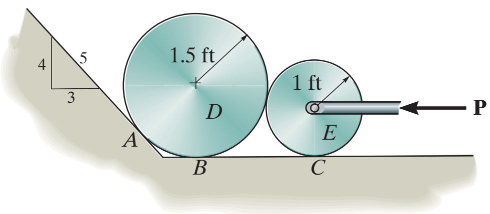
$N' = 204.12$ lb

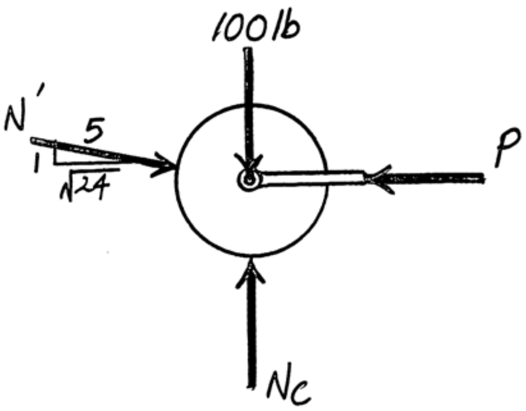
$N_A = 250$ lb **Ans**

$N_B = 9.18$ lb **Ans**

$N_C = 141$ lb **Ans**

5–58. The smooth disks D and E have a weight of 200 lb and 100 lb, respectively. Determine the largest horizontal force P that can be applied to the center of disk E without causing the disk D to move up the incline.





For disk E:

$$\xrightarrow{+} \Sigma F_x = 0; \quad -P + N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad N_C - 100 - N'\left(\frac{1}{5}\right) = 0$$

For disk D:

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_A\left(\frac{4}{5}\right) - N'\left(\frac{\sqrt{24}}{5}\right) = 0$$

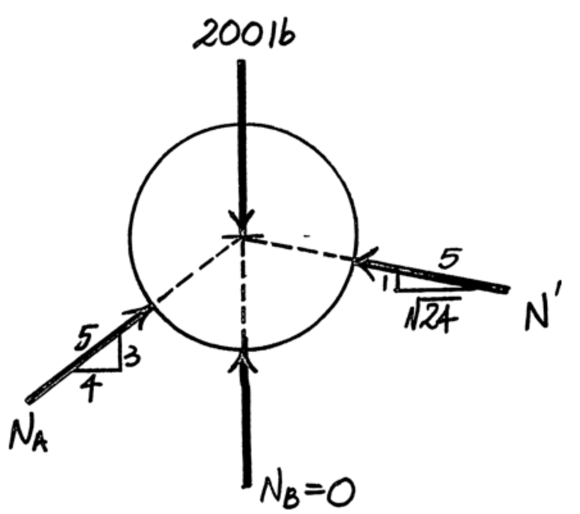
$$+\uparrow \Sigma F_y = 0; \quad N_A\left(\frac{3}{5}\right) + N_B - 200 + N'\left(\frac{1}{5}\right) = 0$$

Require $N_B = 0$ for P_{max}. Solving,

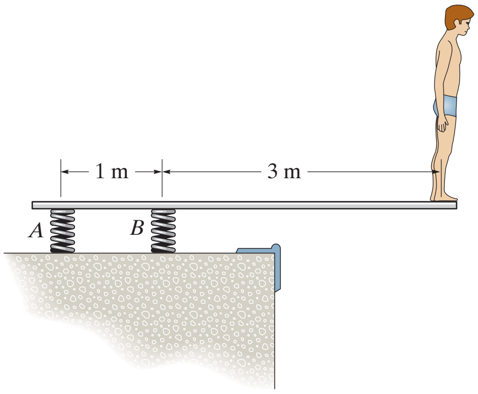
$P_{max} = 210$ lb **Ans**

$N_A = 262$ lb **Ans**

$N_C = 143$ lb **Ans**



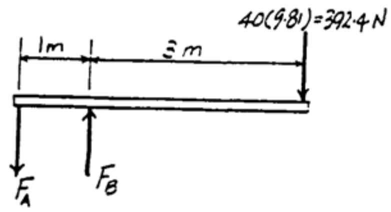
5–59. A man stands out at the end of the diving board, which is supported by two springs A and B, each having a stiffness of $k = 15$ kN/m. In the position shown the board is horizontal. If the man has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.



Equations of Equilibrium : The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\curvearrowleft + \Sigma M_B = 0; \quad F_A(1) - 392.4(3) = 0 \quad F_A = 1177.2 \text{ N}$$

$$\curvearrowleft + \Sigma M_A = 0; \quad F_B(1) - 392.4(4) = 0 \quad F_B = 1569.6 \text{ N}$$

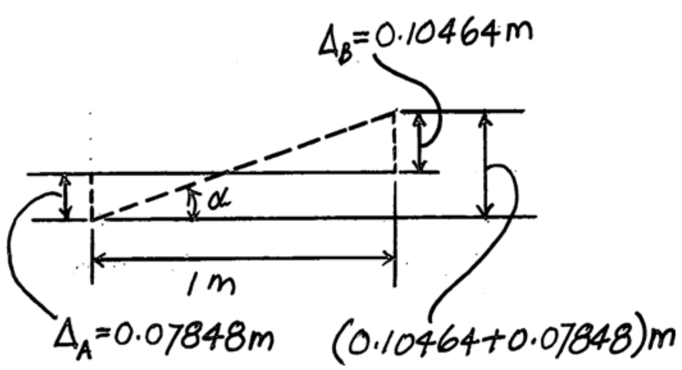


Spring Formula : Applying $\Delta = \frac{F}{k}$, we have

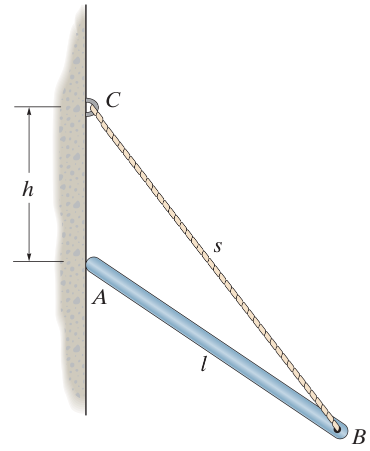
$$\Delta_A = \frac{1177.2}{15(10^3)} = 0.07848 \text{ m} \qquad \Delta_B = \frac{1569.6}{15(10^3)} = 0.10464 \text{ m}$$

Geometry : The angle of tilt α is

$$\alpha = \tan^{-1}\left(\frac{0.10464 + 0.07848}{1}\right) = 10.4° \qquad \textbf{Ans}$$



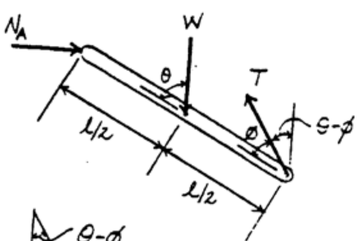
***5–60.** The uniform rod has a length l and weight W. It is supported at one end A by a smooth wall and the other end by a cord of length s which is attached to the wall as shown. Show that for equilibrium it is required that $h = [(s^2 - l^2)/3]^{1/2}$.



Equations of Equilibrium : The tension in the cable can be obtained directly by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \quad T\sin\phi(l) - W\sin\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W\sin\theta}{2\sin\phi}$$

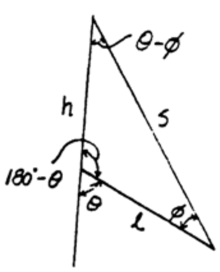


Using the result $T = \dfrac{W\sin\theta}{2\sin\phi}$,

$$+\uparrow \Sigma F_y = 0; \quad \frac{W\sin\theta}{2\sin\phi}\cos(\theta - \phi) - W = 0$$

$$\sin\theta\cos(\theta - \phi) - 2\sin\phi = 0 \qquad [1]$$

Geometry : Applying the sine law with $\sin(180° - \theta) = \sin\theta$, we have



$$\frac{\sin\phi}{h} = \frac{\sin\theta}{s} \qquad \sin\phi = \frac{h}{s}\sin\theta \qquad [2]$$

Substituting Eq. [2] into [1] yields

$$\cos(\theta - \phi) = \frac{2h}{s} \qquad [3]$$

Using the cosine law,

$$l^2 = h^2 + s^2 - 2hs\cos(\theta - \phi)$$

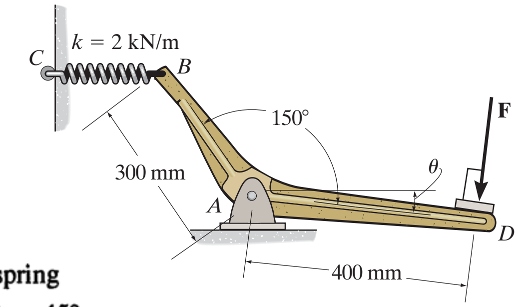
$$\cos(\theta - \phi) = \frac{h^2 + s^2 - l^2}{2hs} \qquad [4]$$

Equating Eqs. [3] and [4] yields

$$\frac{2h}{s} = \frac{h^2 + s^2 - l^2}{2hs}$$

$$h = \sqrt{\frac{s^2 - l^2}{3}} \qquad (Q.E.D)$$

•**5–61.** If spring BC is unstretched with $\theta = 0°$ and the bell crank achieves its equilibrium position when $\theta = 15°$, determine the force **F** applied perpendicular to segment AD and the horizontal and vertical components of reaction at pin A. Spring BC remains in the horizontal postion at all times due to the roller at C.



Spring Force Formula: From the geometry shown in Fig. *a*, the stretch of spring BC when the bell crank rotates $\theta = 15°$ about point A is $x = 0.3\cos 30° - 0.3\cos 45° = 0.04768$ m. Thus, the force developed in spring BC is given by

$$F_{sp} = kx = 2000(0.04768) = 95.35\text{ N}$$

Equations of Equilibrium: From the free - body diagram of the bell crank, Fig. *b*, F can be obtained by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 95.35\sin 45°(300) - F(400) = 0$

$F = 50.57\text{ N} = 50.6\text{ N}$ **Ans.**

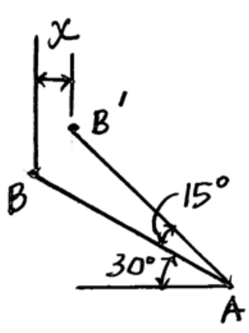
Using the above result and writing the force equations of equilibrium along the x and y axes,

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad A_x - 50.57\sin 15° - 95.35 = 0$

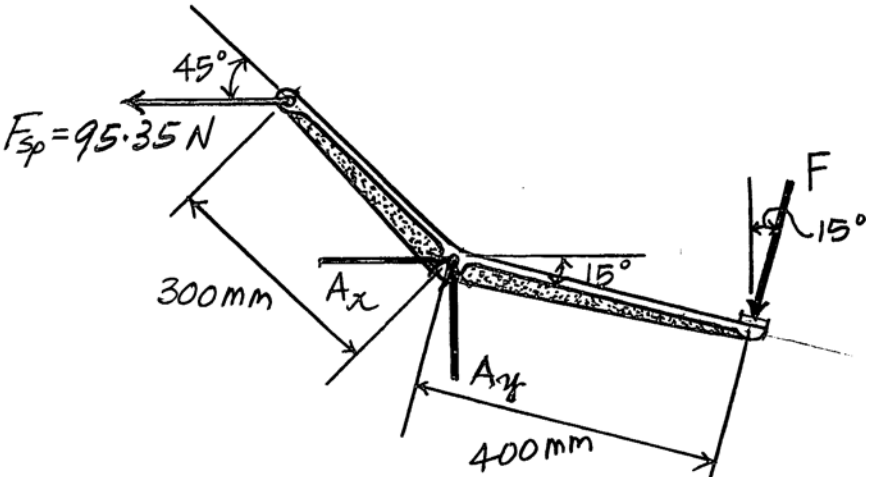
$A_x = 108.44\text{ N} = 108\text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 50.57\cos 15° = 0$

$A_y = 48.84\text{ N} = 48.8\text{ N}$ **Ans.**

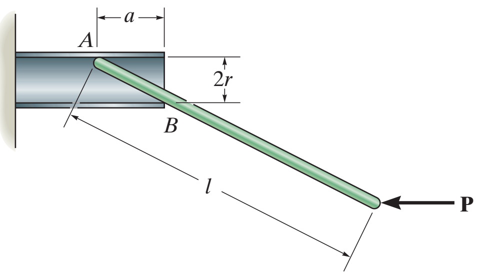


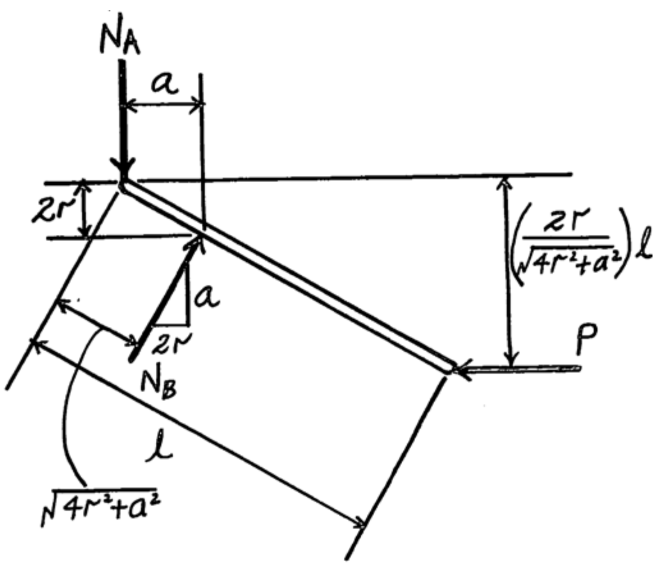
(a)



(b)

5–62. The thin rod of length l is supported by the smooth tube. Determine the distance a needed for equilibrium if the applied load is **P**.





$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{2r}{\sqrt{4r^2 + a^2}} N_B - P = 0$$

$$\circlearrowleft + \Sigma M_A = 0; \quad -P\left(\frac{2r}{\sqrt{4r^2 + a^2}}\right) l + N_B \sqrt{4r^2 + a^2} = 0$$

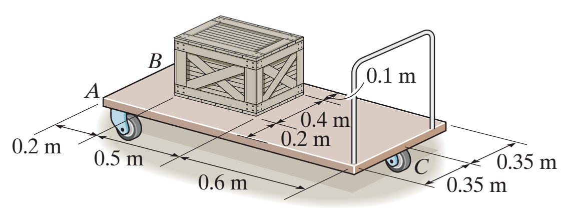
$$\frac{4r^2 l}{4r^2 + a^2} - \sqrt{4r^2 + a^2} = 0$$

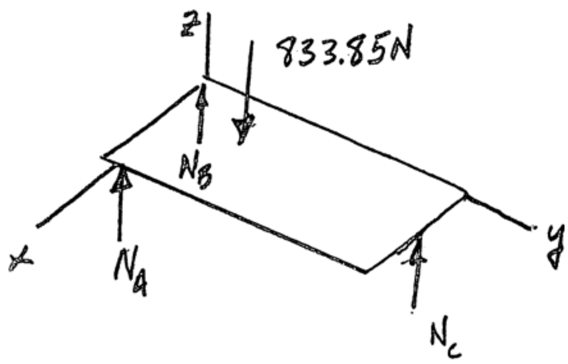
$$4r^2 l = (4r^2 + a^2)^{\frac{3}{2}}$$

$$(4r^2 l)^{\frac{2}{3}} = 4r^2 + a^2$$

$$a = \sqrt{(4r^2 l)^{\frac{2}{3}} - 4r^2} \qquad \textbf{Ans}$$

5–63. The cart supports the uniform crate having a mass of 85 kg. Determine the vertical reactions on the three casters at A, B, and C. The caster at B is not shown. Neglect the mass of the cart.





Equations of Equilibrium : The normal reaction N_C can be obtained directly by summing moments about x axis.

$\Sigma M_x = 0;$ $N_C(1.3) - 833.85(0.45) = 0$

$N_C = 288.64 \text{ N} = 289 \text{ N}$ **Ans**

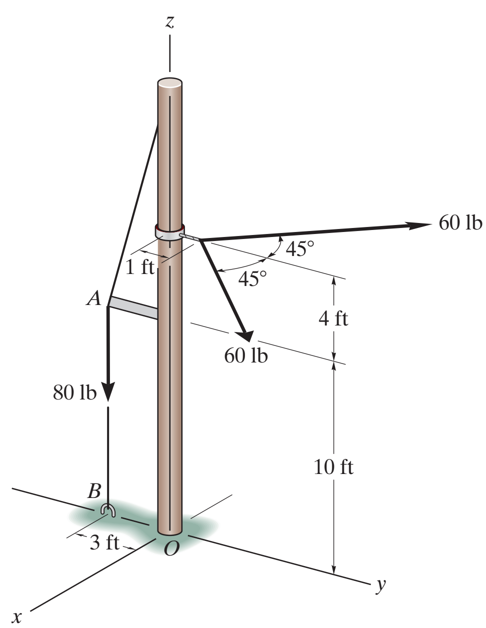
$\Sigma M_y = 0;$ $833.85(0.3) - 288.64(0.35) - N_A(0.7) = 0$

$N_A = 213.04 \text{ N} = 213 \text{ N}$ **Ans**

$\Sigma F_z = 0;$ $N_B + 288.64 + 213.04 - 833.85 = 0$

$N_B = 332 \text{ N}$ **Ans**

***5–64.** The pole for a power line is subjected to the two cable forces of 60 lb, each force lying in a plane parallel to the $x-y$ plane. If the tension in the guy wire AB is 80 lb, determine the x, y, z components of reaction at the fixed base of the pole, O.



Equations of Equilibrium :

$\Sigma F_x = 0;\quad O_x + 60\sin 45° - 60\sin 45° = 0$

$O_x = 0$ **Ans**

$\Sigma F_y = 0;\quad O_y + 60\cos 45° + 60\cos 45° = 0$

$O_y = -84.9\text{ lb}$ **Ans**

$\Sigma F_z = 0;\quad O_z - 80 = 0 \quad O_z = 80.0\text{ lb}$ **Ans**

$\Sigma M_x = 0;\quad (M_O)_x + 80(3) - 2[60\cos 45°(14)] = 0$

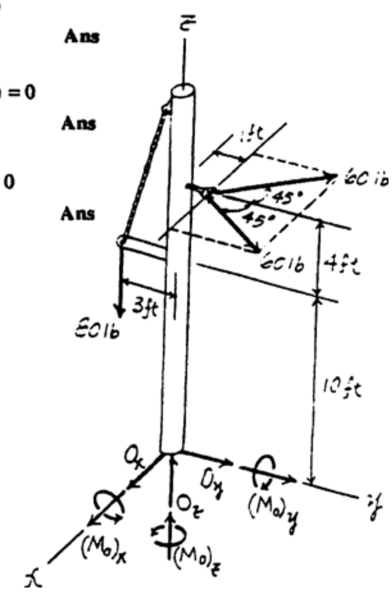
$(M_O)_x = 948\text{ lb}\cdot\text{ft}$ **Ans**

$\Sigma M_y = 0;\quad (M_O)_y + 60\sin 45°(14) - 60\sin 45°(14) = 0$

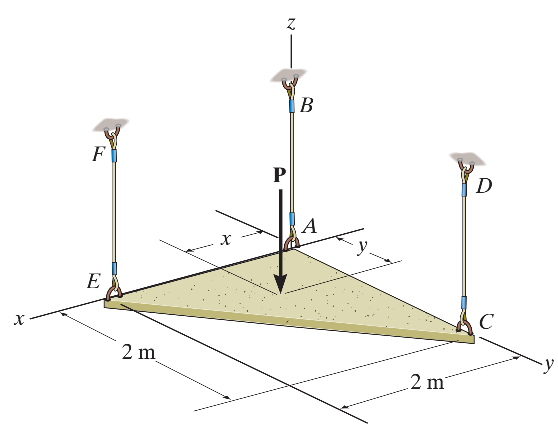
$(M_O)_y = 0$ **Ans**

$\Sigma M_z = 0;\quad (M_O)_z + 60\sin 45°(1) - 60\sin 45°(1) = 0$

$(M_O)_z = 0$ **Ans**



•5–65. If $P = 6$ kN, $x = 0.75$ m and $y = 1$ m, determine the tension developed in cables *AB, CD,* and *EF*. Neglect the weight of the plate.



Equations of Equilibrium: From the free - body diagram, Fig. *a*, T_{CD} and T_{EF} can be obtained by writing the moment equation of equilibrium about the x and y axes, respectively.

$\Sigma M_x = 0;\quad T_{CD}(2) - 6(1) = 0$

$$T_{CD} = 3\text{ kN} \qquad \textbf{Ans.}$$

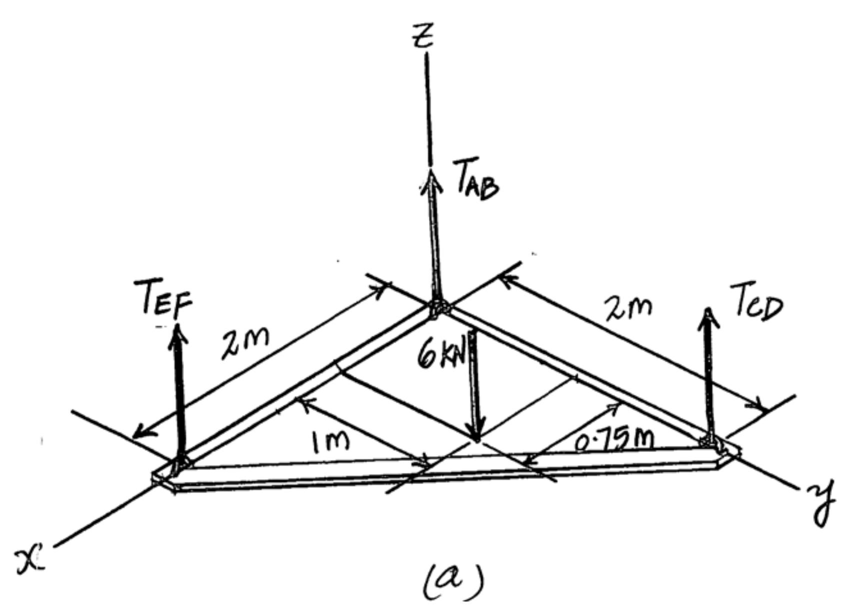
$\Sigma M_y = 0;\quad T_{EF}(2) - 6(0.75) = 0$

$$T_{EF} = 2.25\text{ kN} \qquad \textbf{Ans.}$$

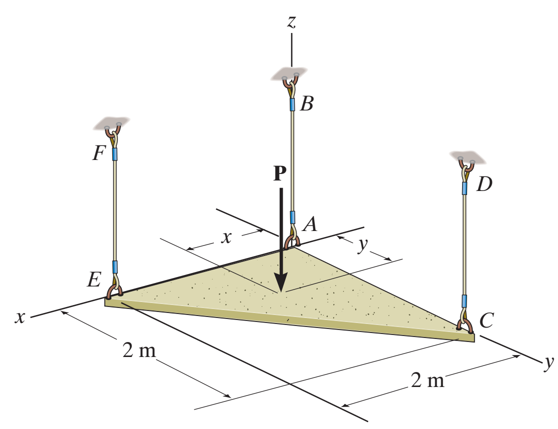
Using the above results and writing the force equation of equilibrium along the z axis,

$\Sigma F_z = 0;\quad T_{AB} + 3 + 2.25 - 6 = 0$

$$T_{AB} = 0.75\text{ kN} \qquad \textbf{Ans.}$$



5–66. Determine the location x and y of the point of application of force **P** so that the tension developed in cables AB, CD, and EF is the same. Neglect the weight of the plate.



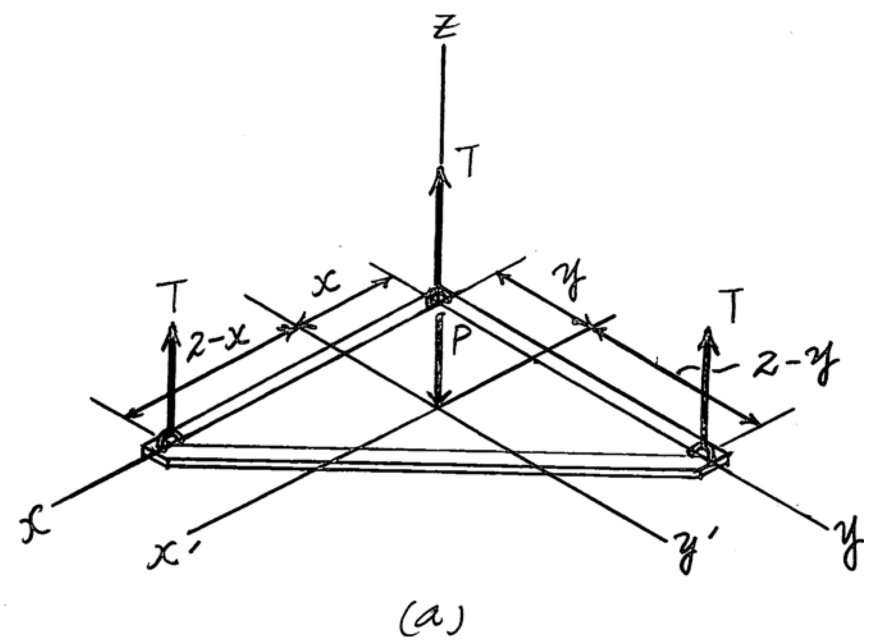
Equations of Equilibrium: From the free - body diagram of the plate, Fig. a, and writing the moment equations of equilibrium about the x' and y' axes,

$\Sigma M_{x'} = 0;$ $\quad T(2-y) - 2T(y) = 0$

$y = 0.667 \text{ m}$ **Ans.**

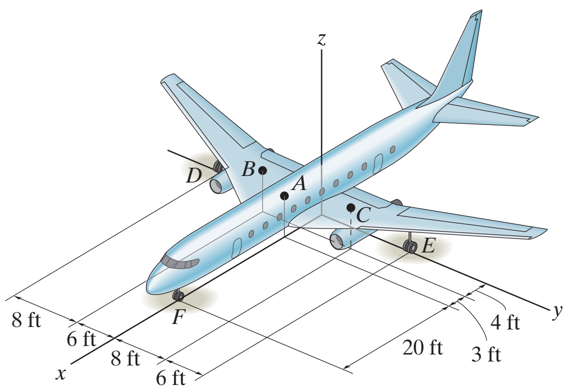
$\Sigma M_{y'} = 0;$ $\quad 2T(x) - T(2-x) = 0$

$x = 0.667 \text{ m}$ **Ans.**



(a)

5–67. Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage A and wings B and C are located as shown. If these components have weights $W_A = 45\,000$ lb, $W_B = 8000$ lb, and $W_C = 6000$ lb, determine the normal reactions of the wheels D, E, and F on the ground.



$\Sigma M_x = 0;\quad 8000(6) - R_D(14) - 6000(8) + R_E(14) = 0$

$\Sigma M_y = 0;\quad 8000(4) + 45\,000(7) + 6000(4) - R_F(27) = 0$

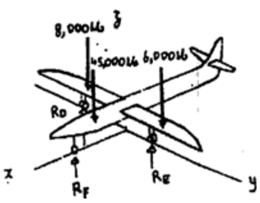
$\Sigma F_z = 0;\quad R_D + R_E + R_F - 8000 - 6000 - 45\,000 = 0$

Solving,

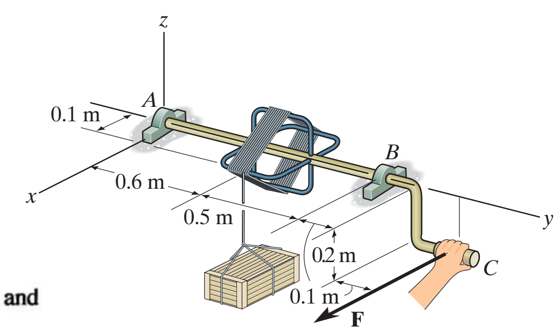
$R_D = 22.6$ kip **Ans**

$R_E = 22.6$ kip **Ans**

$R_F = 13.7$ kip **Ans**



***5–68.** Determine the magnitude of force **F** that must be exerted on the handle at *C* to hold the 75-kg crate in the position shown. Also, determine the components of reaction at the thrust bearing *A* and smooth journal bearing *B*.



Equations of Equilibrium: From the free - body diagram, Fig. *a*, F, B_z, A_z, and A_y can be obtained by writing the moment equation of equilibrium about the y, x, and x' axes and the force equation of equilibrium along the y axis.

$\Sigma M_y = 0;\quad -F(0.2) + 75(9.81)(0.1) = 0$

$F = 367.875 \text{ N} = 368 \text{ N}$ **Ans.**

$\Sigma M_x = 0;\quad B_z(0.5 + 0.6) - 75(9.81)(0.6) = 0$

$B_z = 401.32 \text{ N} = 401 \text{ N}$ **Ans.**

$\Sigma M_{x'} = 0;\quad -A_z(0.6 + 0.5) + 75(9.81)(0.5) = 0$

$A_z = 334.43 \text{ N} = 334 \text{ N}$ **Ans.**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans.**

Using the result $F = 367.875$ N and writing the moment equations of equilibrium about the z and z' axes,

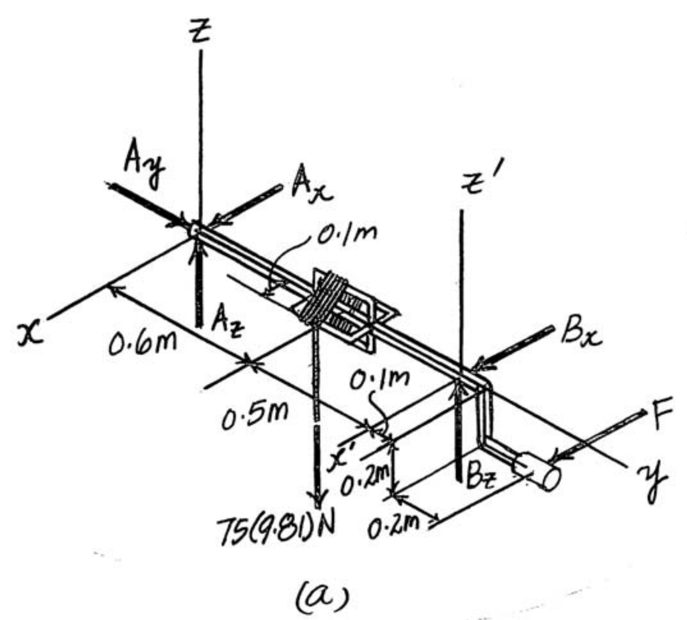
$\Sigma M_z = 0;\quad -B_x(0.5 + 0.6) - 367.875(0.2 + 0.1 + 0.5 + 0.6) = 0$

$B_x = -468.20 \text{ N} = -468 \text{ N}$ **Ans.**

$\Sigma M_{z'} = 0;\quad A_x(0.6 + 0.5) - 367.875(0.2 + 0.1) = 0$

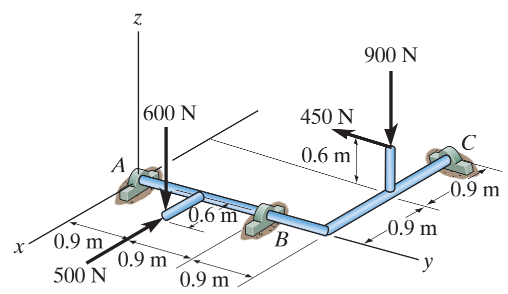
$A_x = 100.33 \text{ N} = 100 \text{ N}$ **Ans.**

The negative signs indicate that $\mathbf{B}_x$ act in the opposite sense to that shown on the free - body diagram.



(a)

•**5–69.** The shaft is supported by three smooth journal bearings at *A*, *B*, and *C*. Determine the components of reaction at these bearings.



Equations of Equilibrium: From the free - body diagram, Fig. *a*, C_y and C_z can be obtained by writing the force equation of equilibrium along the *y* axis and the moment equation of equilibrium about the *y* axis.

$\Sigma F_y = 0;\quad C_y - 450 = 0$

$C_y = 450\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad C_z(0.9+0.9) - 900(0.9) + 600(0.6) = 0$

$C_z = 250\text{ N}$ **Ans.**

Using the above results

$\Sigma M_x = 0;\quad B_z(0.9+0.9) + 250(0.9+0.9+0.9) + 450(0.6) - 900(0.9+0.9+0.9) - 600(0.9) = 0$

$B_z = 1125\text{ N} = 1.125\text{ kN}$ **Ans.**

$\Sigma M_{x'} = 0;\quad 600(0.9) + 450(0.6) - 900(0.9) + 250(0.9) - A_z(0.9+0.9) = 0$

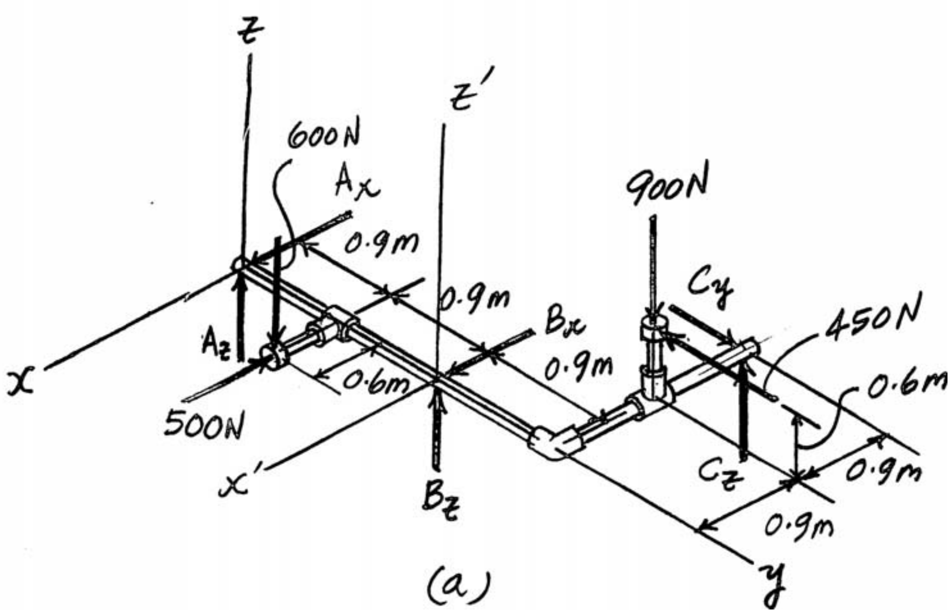
$A_z = 125\text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad -B_x(0.9+0.9) + 500(0.9) + 450(0.9) - 450(0.9+0.9) = 0$

$B_x = 25\text{ N}$ **Ans.**

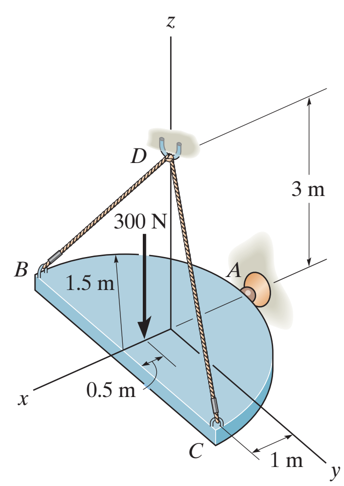
$\Sigma F_x = 0;\quad A_x + 25 - 500 = 0$

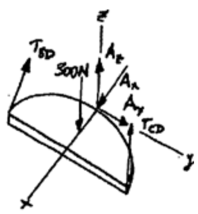
$A_x = 475\text{ N}$ **Ans.**



(a)

5–70. Determine the tension in cables *BD* and *CD* and the *x*, *y*, *z* components of reaction at the ball-and-socket joint at *A*.





$\mathbf{r}_{BD} = \{-1\,\mathbf{i} + 1.5\,\mathbf{j} + 3\,\mathbf{k}\}$ m; $\quad r_{BD} = 3.50$ m

$$\mathbf{T}_{BD} = T_{BD}\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = -0.2857\,T_{BD}\,\mathbf{i} + 0.4286\,T_{BD}\,\mathbf{j} + 0.8571\,T_{BD}\,\mathbf{k}$$

In a similar manner,

$$\mathbf{T}_{CD} = T_{CD}\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = -0.2857\,T_{CD}\,\mathbf{i} - 0.4286\,T_{CD}\,\mathbf{j} + 0.8571\,T_{CD}\,\mathbf{k}$$

Thus, using the components of $\mathbf{T}_{BD}$ and $\mathbf{T}_{CD}$, the scalar equations of equilibrium become :

$\Sigma F_x = 0; \quad A_x - 0.2857\,T_{BD} - 0.2857\,T_{CD} = 0$

$\Sigma F_y = 0; \quad A_y + 0.4286\,T_{BD} - 0.4286\,T_{CD} = 0$

$\Sigma F_z = 0; \quad A_z + 0.8571\,T_{BD} + 0.8571\,T_{CD} - 300 = 0$

$\Sigma M_{Ax} = 0; \quad -(0.8571\,T_{BD})(1.5) + (0.8571\,T_{CD})(1.5) = 0$

$\Sigma M_{Ay} = 0; \quad 300(1) - (0.8571\,T_{BD})(1.5) - (0.8571\,T_{CD})(1.5) = 0$

Solving

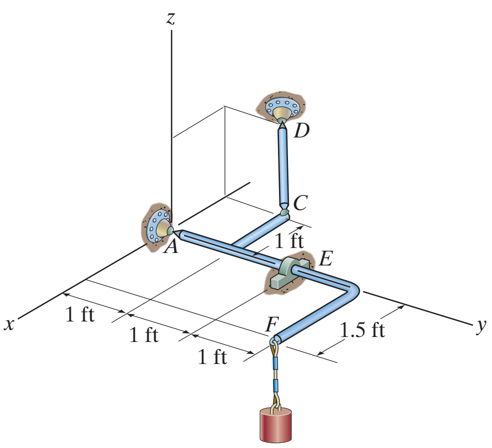
$T_{BD} = T_{CD} = 117$ N **Ans**

$A_x = 66.7$ N **Ans**

$A_y = 0$ **Ans**

$A_z = 100$ N **Ans**

5–71. The rod assembly is used to support the 250-lb cylinder. Determine the components of reaction at the ball-and-socket joint A, the smooth journal bearing E, and the force developed along rod CD. The connections at C and D are ball-and-socket joints.



Equations of Equilibrium: Since rod CD is a two - force member, it exerts a force $\mathbf{F}_{DC}$ directed along its axis as defined by $\mathbf{u}_{DC}$ on rod BC, Fig. a. Expressing each of the forces indicated on the free - body diagram in Cartesian vector form,

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$
$$\mathbf{F}_E = E_x\mathbf{i} + E_z\mathbf{k}$$
$$\mathbf{W} = [-250\mathbf{k}]\text{ lb}$$
$$\mathbf{F}_{DC} = -F_{DC}\,\mathbf{k}$$

Applying the force equation of equilibrium

$\Sigma\mathbf{F} = \mathbf{0};\quad \mathbf{F}_A + \mathbf{F}_E + \mathbf{F}_{DC} + \mathbf{W} = \mathbf{0}$

$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + (E_x\mathbf{i} + E_z\mathbf{k}) + (-F_{DC}\,\mathbf{k}) + (-250\mathbf{k}) = \mathbf{0}$$

$$(A_x + E_x)\mathbf{i} + (A_y)\mathbf{j} + (A_z + E_z - F_{DC})\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$A_x + E_x = 0$ (1)

$A_y = 0$ (2)

$A_z + E_z - F_{DC} - 250 = 0$ (3)

In order to apply the moment equation of equilibrium about point A, the position vectors $\mathbf{r}_{AC}$, $\mathbf{r}_{AE}$, and $\mathbf{r}_{AF}$, Fig. a, must be determined first.

$$\mathbf{r}_{AC} = [-1\mathbf{i} + 1\mathbf{j}]\text{ft}$$
$$\mathbf{r}_{AE} = [2\mathbf{j}]\text{ft}$$
$$\mathbf{r}_{AF} = [1.5\mathbf{i} + 3\mathbf{j}]\text{ft}$$

Thus,

$\Sigma\mathbf{M}_A = \mathbf{0};\ (\mathbf{r}_{AC} \times \mathbf{F}_{DC}) + (\mathbf{r}_{AE} \times \mathbf{F}_E) + (\mathbf{r}_{AF} \times \mathbf{W}) = 0$

$$(-1\mathbf{i} + 1\mathbf{j}) \times (-F_{DC}\,\mathbf{k}) + (2\mathbf{j}) \times (E_x\mathbf{i} + E_z\mathbf{k}) + (1.5\mathbf{i} + 3\mathbf{j}) \times (-250\mathbf{k}) = 0$$

$$(-F_{DC} + 2E_z - 750)\mathbf{i} + (375 - F_{DC})\mathbf{j} + (-2E_x)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$-F_{DC} + 2E_z - 750 = 0$ (4)

$375 - F_{DC} = 0$ (5)

$-2E_x = 0$ (6)

Solving Eqs. (1) through (6), yields

$F_{DC} = 375 \text{ lb}$ **Ans.**

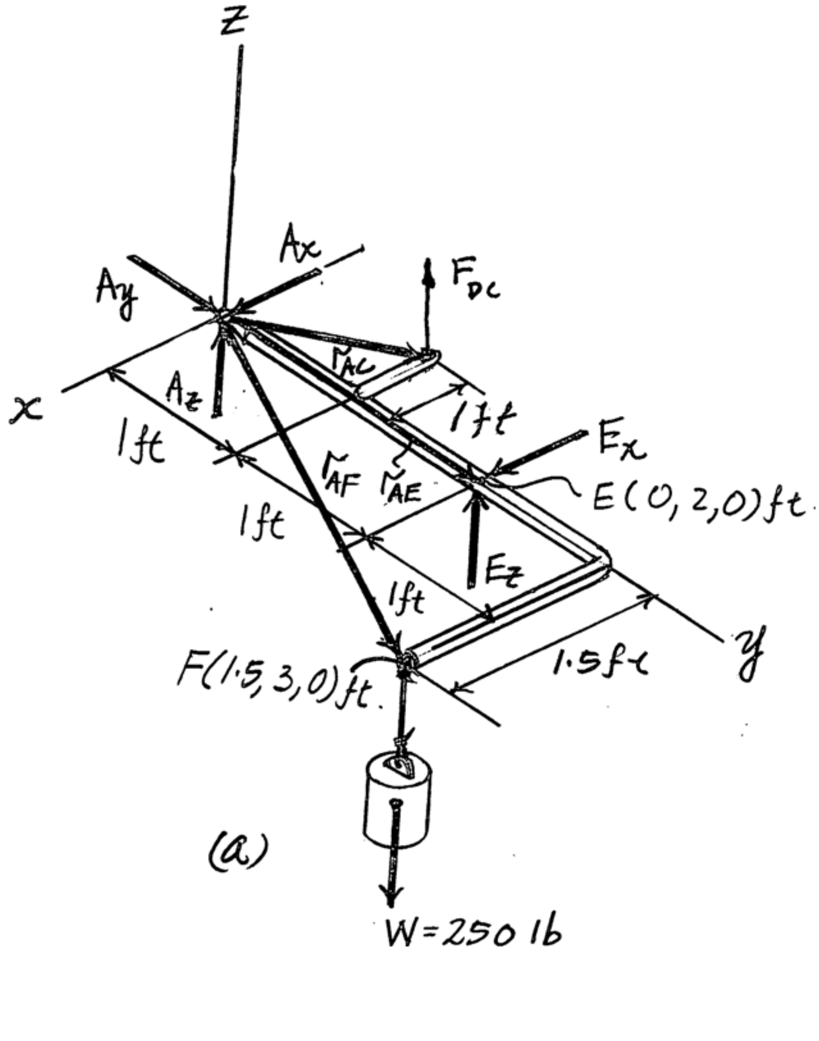
$E_x = 0$ **Ans.**

$E_z = 562.5 \text{ lb}$ **Ans.**

$A_x = 0$ **Ans.**

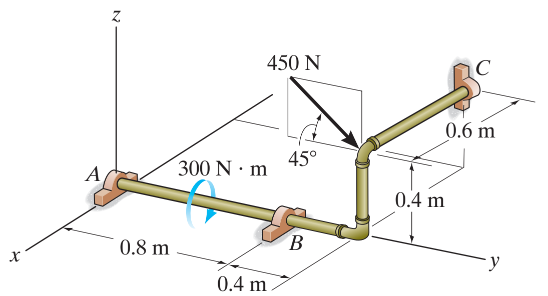
$A_y = 0$ **Ans.**

$A_z = 62.5 \text{ lb}$ **Ans.**



(a)

***5–72.** Determine the components of reaction acting at the smooth journal bearings A, B, and C.



Equations of Equilibrium: From the free - body diagram of the shaft, Fig. *a*, C_y and C_z can be obtained by writing the force equation of equilibrium along the y axis and the moment equation of equilibrium about the y axis.

$\Sigma F_y = 0;\quad 450\cos 45° + C_y = 0$

$C_y = -318.20\text{ N} = -318\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad C_z(0.6) - 300 = 0$

$C_z = 500\text{ N}$ **Ans.**

Using the above results and writing the moment equations of equilibrium about the x and z axes,

$\Sigma M_x = 0;\quad B_z(0.8) - 450\cos 45°(0.4) - 450\sin 45°(0.8+0.4) + (318.20)(0.4) + 500(0.8+0.4) = 0$

$B_z = -272.70\text{ N} = -273\text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad -B_x(0.8) - (-318.20)(0.6) = 0$

$B_x = 238.65\text{ N} = 239\text{ N}$ **Ans.**

Finally, using the above results and writing the force equations of equilibrium along the x and y axes,

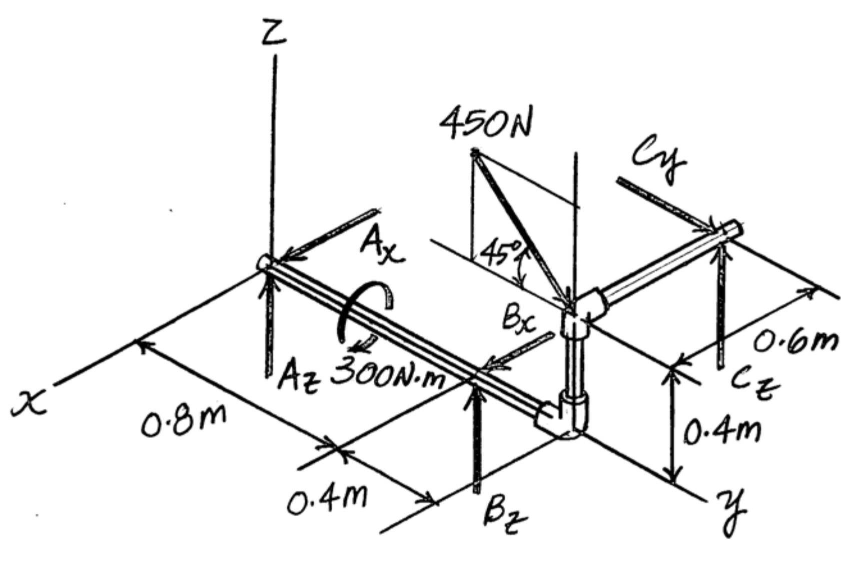
$\Sigma F_x = 0;\quad A_x + 238.5 = 0$

$A_x = -238.65\text{ N} = -239\text{ N}$ **Ans.**

$\Sigma F_z = 0;\quad A_z - (-272.70) + 500 - 450\sin 45° = 0$

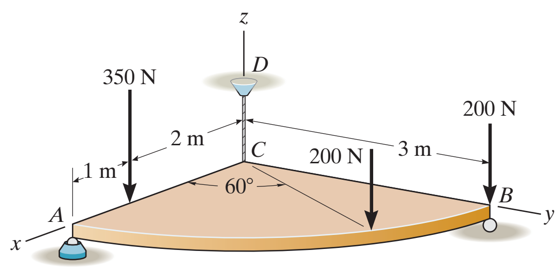
$A_z = 90.90\text{ N} = 90.9\text{ N}$ **Ans.**

The negative signs indicate that $\mathbf{C}_y$, $\mathbf{B}_z$ and $\mathbf{A}_x$ act in the opposite sense of that shown on the free - body diagram.



(a)

•5–73. Determine the force components acting on the ball-and-socket at A, the reaction at the roller B and the tension on the cord CD needed for equilibrium of the quarter circular plate.



Equations of Equilibrium : The normal reaction N_B and A_z can be obtained directly by summing moments about the x and y axes respectively.

$\Sigma M_x = 0;\quad N_B(3) - 200(3) - 200(3\sin 60°) = 0$

$N_B = 373.21\text{ N} = 373\text{ N}$ **Ans**

$\Sigma M_y = 0;\quad 350(2) + 200(3\cos 60°) - A_z(3) = 0$

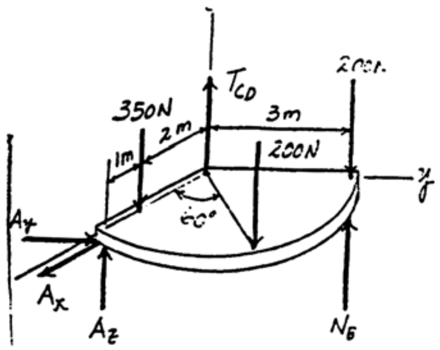
$A_z = 333.33\text{ N} = 333\text{ N}$ **Ans**

$\Sigma F_z = 0;\quad T_{CD} + 373.21 + 333.33 - 350 - 200 - 200 = 0$

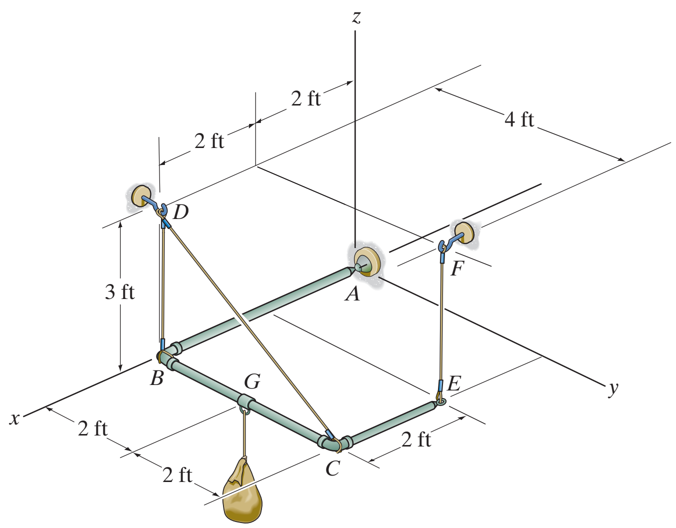
$T_{CD} = 43.5\text{ N}$ **Ans**

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans**



5–74. If the load has a weight of 200 lb, determine the *x, y, z* components of reaction at the ball-and-socket joint *A* and the tension in each of the wires.



Equations of Equilibrium: Expressing the forces indicated on the free - body diagram, Fig. *a*, in Cartesian vector form,

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$
$$\mathbf{W} = [-200\mathbf{k}]\ \text{lb}$$
$$\mathbf{F}_{BD} = F_{BD}\mathbf{k}$$
$$\mathbf{F}_{CD} = F_{CD}\mathbf{u}_{CD} = F_{CD}\left[\frac{(4-4)\mathbf{i} + (0-4)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(4-4)^2 + (0-4)^2 + (3-0)^2}}\right] = \left(-\frac{4}{5}F_{CD}\mathbf{j} + \frac{3}{5}F_{CD}\mathbf{k}\right)$$
$$\mathbf{F}_{EF} = F_{EF}\mathbf{k}$$

Applying the force equation of equilibrium,

$$\Sigma\mathbf{F} = \mathbf{0}; \quad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{CD} + \mathbf{F}_{EF} + \mathbf{W} = \mathbf{0}$$
$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + F_{BD}\mathbf{k} + \left(-\frac{4}{5}F_{CD}\mathbf{j} + \frac{3}{5}F_{CD}\mathbf{k}\right) + F_{EF}\mathbf{k} + (-200\mathbf{k}) = \mathbf{0}$$
$$(A_x)\mathbf{i} + \left(A_y - \frac{4}{5}F_{CD}\right)\mathbf{j} + \left(A_z + F_{BD} + \frac{3}{5}F_{CD} + F_{EF} - 200\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$$A_x = 0 \qquad (1)$$
$$A_y - \frac{4}{5}F_{CD} = 0 \qquad (2)$$
$$A_z + F_{BD} + \frac{3}{5}F_{CD} + F_{EF} - 200 = 0 \qquad (3)$$

In order to write the moment equation of equilibrium about point *A*, the position vectors $\mathbf{r}_{AB}$, $\mathbf{r}_{AG}$, $\mathbf{r}_{AC}$, and $\mathbf{r}_{AE}$ must be determined first.

$$\mathbf{r}_{AB} = [4\mathbf{i}]\ \text{ft}$$
$$\mathbf{r}_{AG} = [4\mathbf{i} + 2\mathbf{j}]\ \text{ft}$$
$$\mathbf{r}_{AC} = [4\mathbf{i} + 4\mathbf{j}]\ \text{ft}$$
$$\mathbf{r}_{AE} = [2\mathbf{i} + 4\mathbf{j}]\ \text{ft}$$

Thus,

$$\Sigma\mathbf{M}_A = \mathbf{0}; \ (\mathbf{r}_{AB} \times \mathbf{F}_{BD}) + (\mathbf{r}_{AC} \times \mathbf{F}_{CD}) + (\mathbf{r}_{AE} \times \mathbf{F}_{EF}) + (\mathbf{r}_{AG} \times \mathbf{W}) = \mathbf{0}$$
$$(4\mathbf{i}) \times (F_{BD}\mathbf{k}) + (4\mathbf{i} + 4\mathbf{j}) \times \left(-\frac{4}{5}F_{CD}\mathbf{j} + \frac{3}{5}F_{CD}\mathbf{k}\right) + (2\mathbf{i} + 4\mathbf{j}) \times (F_{EF}\mathbf{k}) + (4\mathbf{i} + 2\mathbf{j}) \times (-200\mathbf{k})$$
$$\left(\frac{12}{5}F_{CD} + 4F_{EF} - 400\right)\mathbf{i} + \left(-4F_{BD} - \frac{12}{5}F_{CD} - 2F_{EF} + 800\right)\mathbf{j} + \left(-\frac{16}{5}F_{CD}\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components,

$$\frac{12}{5}F_{CD} + 4F_{EF} - 400 = 0 \quad (4)$$

$$-4F_{BD} - \frac{12}{15}F_{CD} - 2F_{EF} + 800 = 0 \quad (5)$$

$$-\frac{16}{5}F_{CD} = 0 \quad (6)$$

Solving Eqs. (1) through (6),

$F_{CD} = 0$ **Ans.**

$F_{EF} = 100$ lb **Ans.**

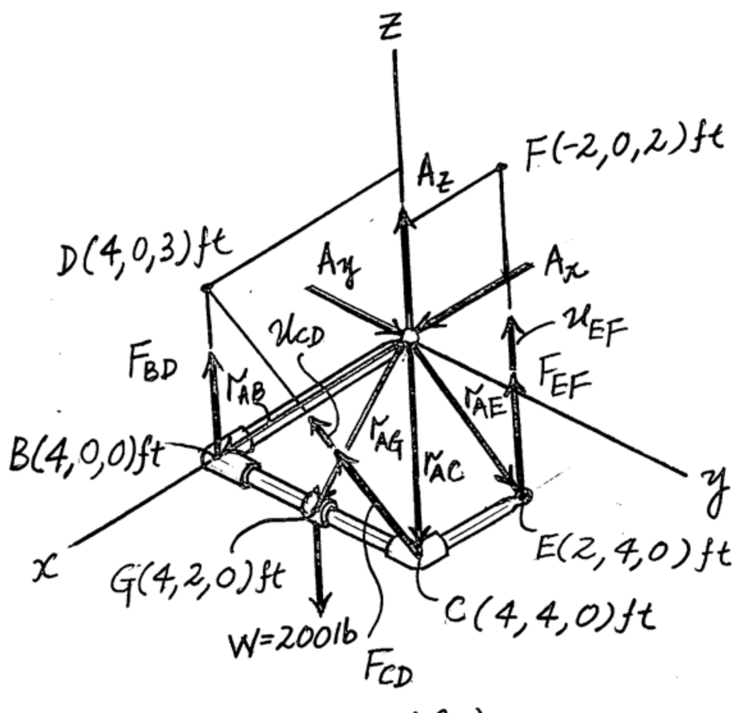
$F_{BD} = 150$ lb **Ans.**

$A_x = 0$ **Ans.**

$A_y = 0$ **Ans.**

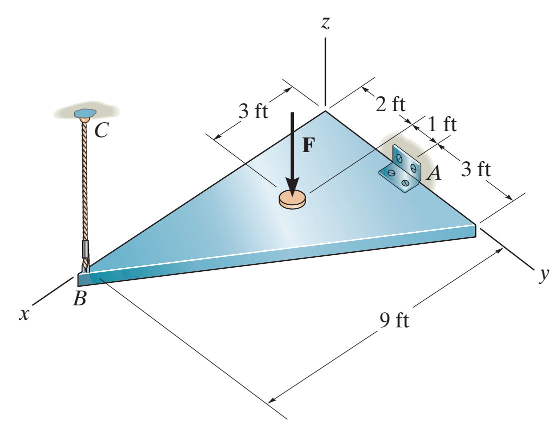
$A_z = 100$ lb **Ans.**

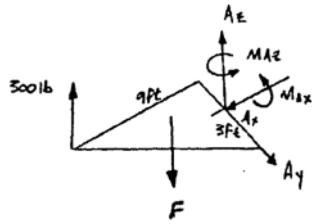
The negative signs indicate that $\mathbf{A}_z$ acts in the opposite sense to that on the free-body diagram.



(a)

5–75. If the cable can be subjected to a maximum tension of 300 lb, determine the maximum force F which may be applied to the plate. Compute the x, y, z components of reaction at the hinge A for this loading.





$\Sigma M_y = 0; \quad 3(F) - 300(9) = 0$

$F = 900 \text{ lb}$ **Ans**

$\Sigma F_x = 0; \quad A_x = 0$ **Ans**

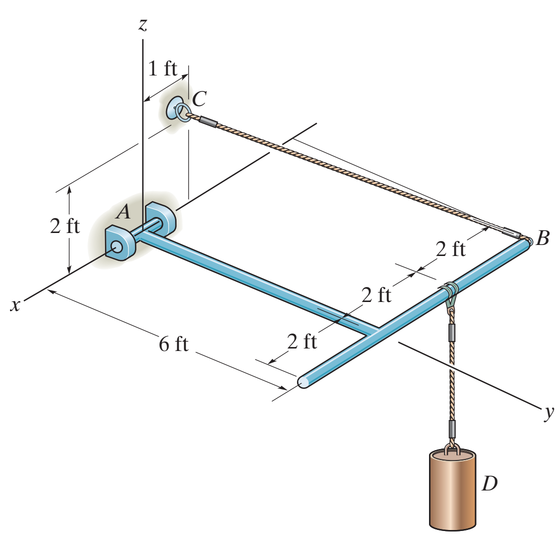
$\Sigma F_y = 0; \quad A_y = 0$ **Ans**

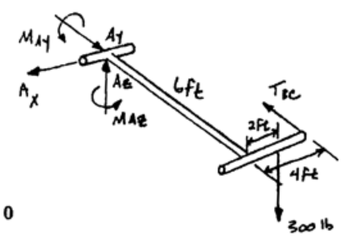
$\Sigma F_z = 0; \quad -900 + 300 + A_z = 0; \quad A_z = 600 \text{ lb}$ **Ans**

$\Sigma M_{Ax} = 0; \quad M_{Ax} + 900(1) - 3(300) = 0; \quad M_{Ax} = 0$ **Ans**

$\Sigma M_{Az} = 0; \quad M_{Az} = 0$ **Ans**

***5–76.** The member is supported by a pin at A and a cable BC. If the load at D is 300 lb, determine the x, y, z components of reaction at the pin A and the tension in cable BC.





$\mathbf{T}_{BC} = T_{BC}\left\{\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right\} \text{ ft}$

$\Sigma F_x = 0; \quad A_x + \left(\frac{3}{7}\right)T_{BC} = 0$

$\Sigma F_y = 0; \quad A_y - \left(\frac{6}{7}\right)T_{BC} = 0$

$\Sigma F_z = 0; \quad A_z - 300 + \left(\frac{2}{7}\right)T_{BC} = 0$

$\Sigma M_x = 0; \quad -300(6) + \left(\frac{2}{7}\right)T_{BC}(6) = 0$

$\Sigma M_y = 0; \quad M_{Ay} - 300(2) + \left(\frac{2}{7}\right)T_{BC}(4) = 0$

$\Sigma M_z = 0; \quad M_{Az} - \left(\frac{3}{7}\right)T_{BC}(6) + \left(\frac{6}{7}\right)T_{BC}(4) = 0$

Solving,

$T_{BC} = 1.05 \text{ kip}$ **Ans**

$A_x = -450 \text{ lb}$ **Ans**

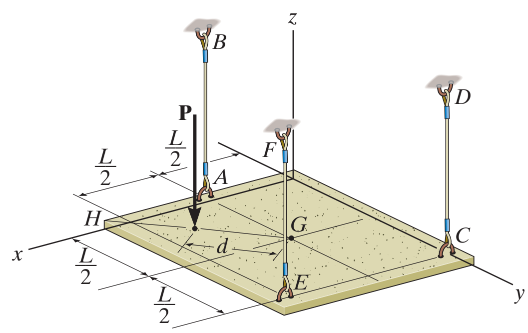
$A_y = 900 \text{ lb}$ **Ans**

$A_z = 0$ **Ans**

$M_{Ay} = -600 \text{ lb} \cdot \text{ft}$ **Ans**

$M_{Az} = -900 \text{ lb} \cdot \text{ft}$ **Ans**

•5–77. The plate has a weight of W with center of gravity at G. Determine the distance d along line GH where the vertical force $P = 0.75W$ will cause the tension in wire CD to become zero.



Equations of Equilibrium: From the free - body diagram, Fig. *a*,

$$\Sigma M_x = 0;\quad T_{EF}(L) - W\left(\frac{L}{2}\right) - 0.75W\left(\frac{L}{2} - d\cos 45°\right) = 0$$

$$T_{EF}L - 0.875WL + 0.5303Wd = 0 \qquad (1)$$

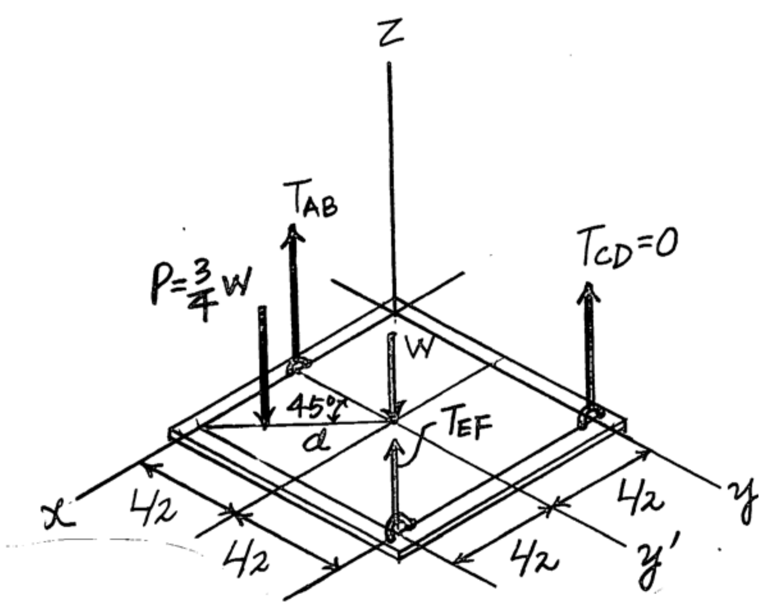
$$\Sigma M_{y'} = 0;\quad 0.75W(d\sin 45°) - T_{EF}\left(\frac{L}{2}\right) = 0$$

$$1.0607Wd - T_{EF}L = 0 \qquad (2)$$

Solving Eqs. (1) and (2) yields

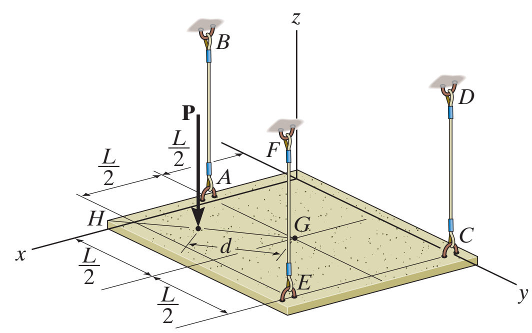
$d = 0.550L$ **Ans.**

$T_{EF} = 0.583W$ **Ans.**



(a)

5–78. The plate has a weight of W with center of gravity at G. Determine the tension developed in wires AB, CD, and EF if the force $P = 0.75W$ is applied at $d = L/2$.



Equations of Equilibrium: From the free-body diagram, Fig. a, T_{AB} can be obtained by writing the moment equation of equilibrium about the x' axis.

$\Sigma M_{x'} = 0;$ $$0.75W\left[\frac{L}{2}+\frac{L}{2}\cos 45°\right]+W\left(\frac{L}{2}\right)-T_{AB}(L)=0$$

$$T_{AB} = 1.1402\ W = 1.14\ W \qquad \textbf{Ans.}$$

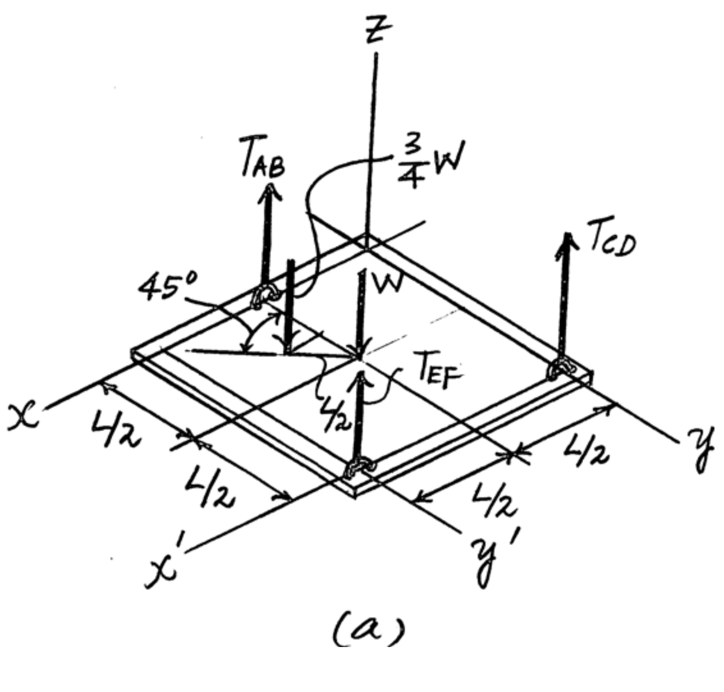
Using the above result and writing the moment equations of equilibrium about the y and y' axes,

$\Sigma M_y = 0;$ $$W\left(\frac{L}{2}\right)+0.75W\left[\frac{L}{2}+\frac{L}{2}\sin 45°\right]-1.1402W\left(\frac{L}{2}\right)-T_{EF}(L)=0$$

$$T_{EF} = 0.570\ W \qquad \textbf{Ans.}$$

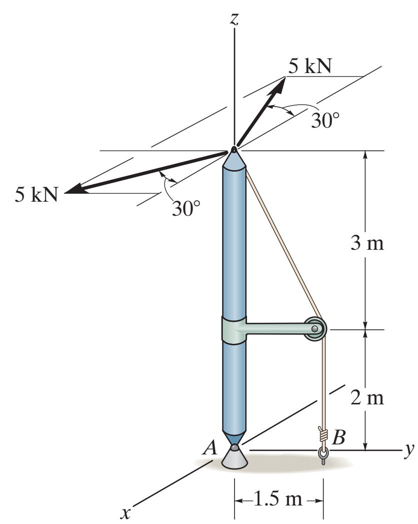
$\Sigma M_{y'} = 0;$ $$T_{CD}(L)+1.1402W\left(\frac{L}{2}\right)-W\left(\frac{L}{2}\right)-0.75W\left[\frac{L}{2}-\frac{L}{2}\sin 45°\right]=0$$

$$T_{CD} = 0.0398\ W \qquad \textbf{Ans.}$$

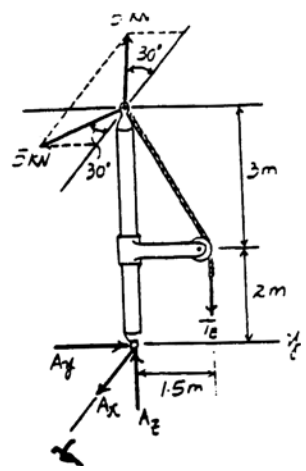


(a)

5–79. The boom is supported by a ball-and-socket joint at A and a guy wire at B. If the 5-kN loads lie in a plane which is parallel to the x–y plane, determine the x, y, z components of reaction at A and the tension in the cable at B.



Equations of Equilibrium :



$\Sigma M_x = 0;\quad 2[5\sin 30°(5)] - T_B(1.5) = 0$

$T_B = 16.67\text{ kN} = 16.7\text{ kN}$ **Ans**

$\Sigma M_y = 0;\quad 5\cos 30°(5) - 5\cos 30°(5) = 0$ ***(Statisfied!)***

$\Sigma F_x = 0;\quad A_x + 5\cos 30° - 5\cos 30° = 0$

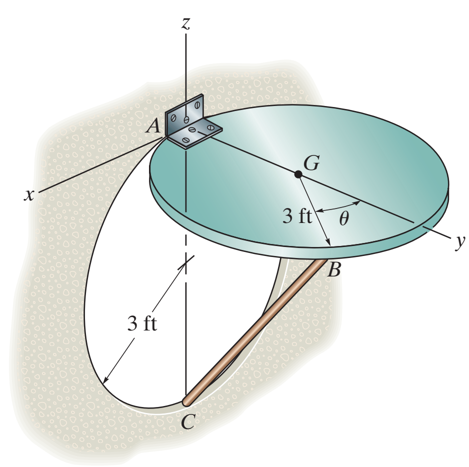
$A_x = 0$ **Ans**

$\Sigma F_y = 0;\quad A_y - 2(5\sin 30°) = 0$

$A_y = 5.00\text{ kN}$ **Ans**

$\Sigma F_z = 0;\quad A_z - 16.67 = 0 \quad A_z = 16.7\text{ kN}$ **Ans**

***5–80.** The circular door has a weight of 55 lb and a center of gravity at G. Determine the x, y, z components of reaction at the hinge A and the force acting along strut CB needed to hold the door in equilibrium. Set $\theta = 45^\circ$.



$$\mathbf{r}_{CB} = 3\sin 45^\circ\,\mathbf{i} + (3 + 3\cos 45^\circ)\,\mathbf{j} + 6\,\mathbf{k}$$

$$= \{2.121\,\mathbf{i} + 5.121\,\mathbf{j} + 6\,\mathbf{k}\}\ \text{ft}$$

$$r_{CB} = \sqrt{(2.121)^2 + (5.121)^2 + (6)^2} = 8.169$$

$$\Sigma F_x = 0; \quad A_x + \left(\frac{2.121}{8.169}\right)F_{CB} = 0$$

$$\Sigma F_y = 0; \quad A_y + \left(\frac{5.121}{8.169}\right)F_{CB} = 0$$

$$\Sigma F_z = 0; \quad A_z - 55 + \left(\frac{6}{8.169}\right)F_{CB} = 0$$

$$\Sigma M_x = 0; \quad -55(3) + \left(\frac{6}{8.169}\right)F_{CB}(3 + 3\cos 45^\circ) = 0$$

$$F_{CB} = 43.9\ \text{lb} \qquad \textbf{Ans}$$

$$A_x = -11.4\ \text{lb} \qquad \textbf{Ans}$$

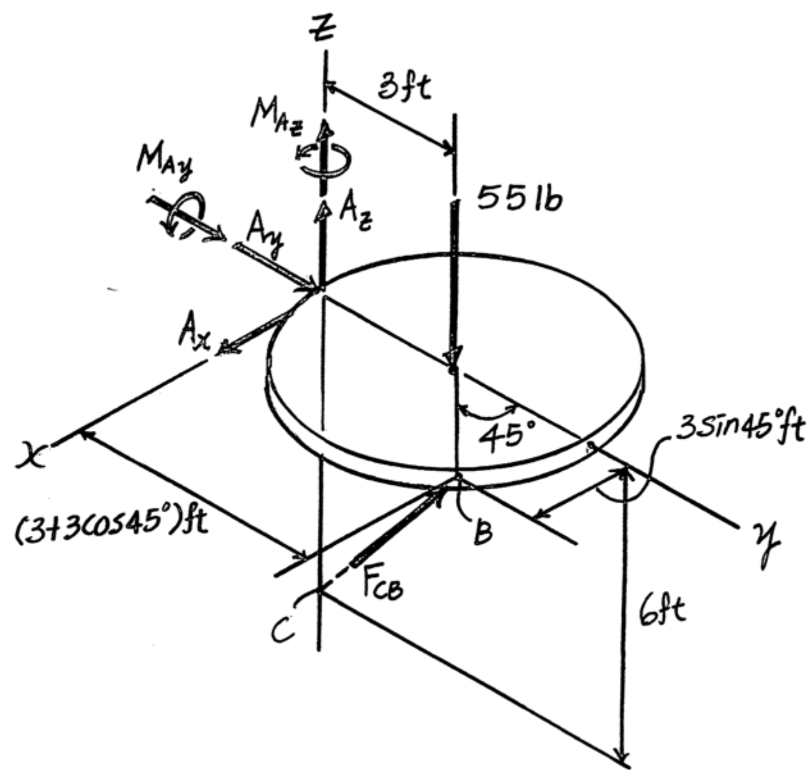
$$A_y = -27.5\ \text{lb} \qquad \textbf{Ans}$$

$$A_z = 22.8\ \text{lb} \qquad \textbf{Ans}$$

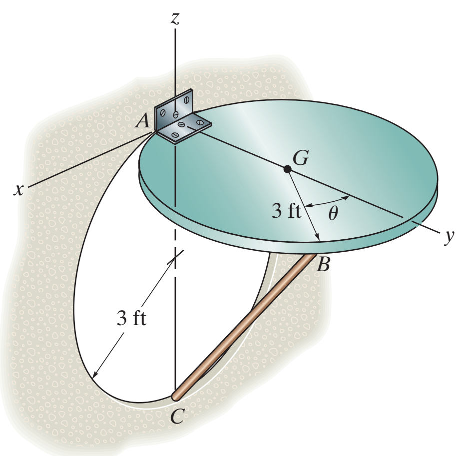
$$\Sigma M_y = 0; \quad M_{Ay} - \left(\frac{6}{8.169}\right)(43.9)(3\sin 45^\circ) = 0; \quad M_{Ay} = 68.3\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$\Sigma M_z = 0; \quad M_{Az} - \left(\frac{2.121}{8.169}\right)(43.9)(3 + 3\cos 45^\circ) + \left(\frac{5.121}{8.169}\right)(43.9)(3\sin 45^\circ) = 0$$

$$M_{Az} = 0 \qquad \textbf{Ans}$$



•5–81. The circular door has a weight of 55 lb and a center of gravity at G. Determine the x, y, z components of reaction at the hinge A and the force acting along strut CB needed to hold the door in equilibrium. Set $\theta = 90°$.



$\mathbf{r}_{CB} = \{3\,\mathbf{i} + 3\,\mathbf{j} + 6\,\mathbf{k}\}$ ft

$r_{CB} = \sqrt{(3)^2 + (3)^2 + (6)^2} = \sqrt{54}$

$\Sigma F_x = 0;\quad A_x + \left(\frac{3}{\sqrt{54}}\right)F_{CB} = 0$

$\Sigma F_y = 0;\quad A_y + \left(\frac{3}{\sqrt{54}}\right)F_{CB} = 0$

$\Sigma F_z = 0;\quad A_z - 55 + \left(\frac{6}{\sqrt{54}}\right)F_{CB} = 0$

$\Sigma M_x = 0;\quad -55\,(3) + \left(\frac{6}{\sqrt{54}}\right)F_{CB}\,(3) = 0$

$F_{CB} = 67.36 = 67.4$ lb **Ans**

Thus,

$A_x = -27.5$ lb **Ans**

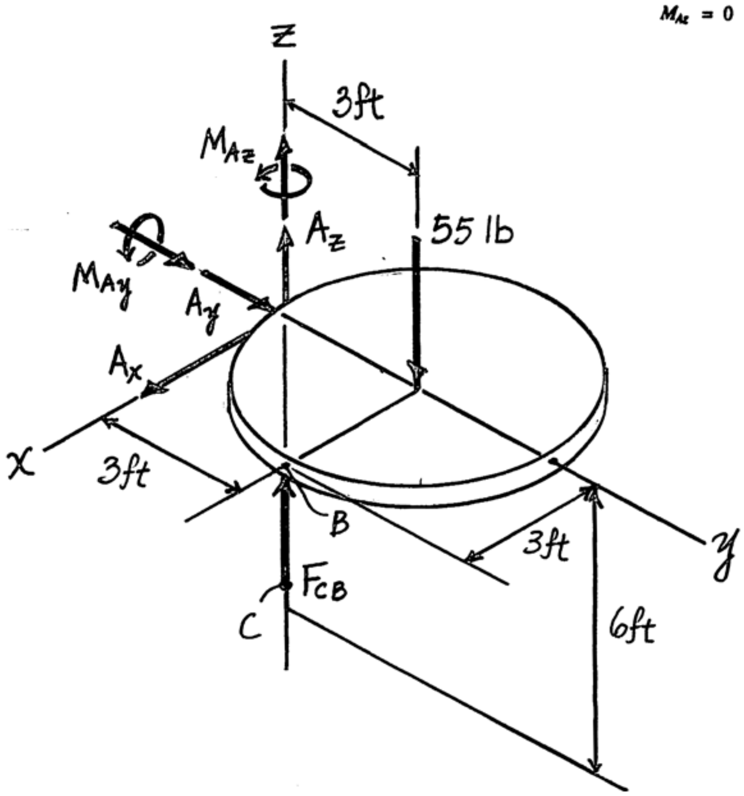
$A_y = -27.5$ lb **Ans**

$A_z = 0$ **Ans**

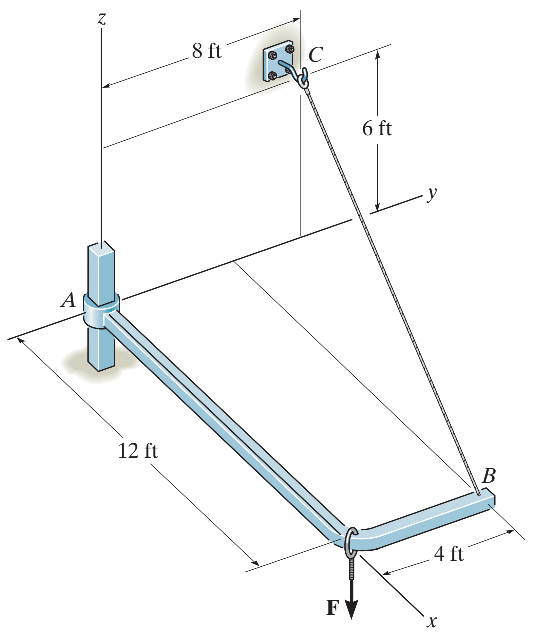
$\Sigma M_y = 0;\quad M_{Ay} - \left(\frac{6}{\sqrt{54}}\right)(67.36)\,(3) = 0;\quad M_{Ay} = 165\ \text{lb}\cdot\text{ft}$ **Ans**

$\Sigma M_z = 0;\quad M_{Az} - \left(\frac{3}{\sqrt{54}}\right)(67.36)\,(3) + \left(\frac{3}{\sqrt{54}}\right)(67.36)\,(3) = 0$

$M_{Az} = 0$ **Ans**



5–82. Member *AB* is supported at *B* by a cable and at *A* by a smooth fixed *square* rod which fits loosely through the square hole of the collar. If $\mathbf{F} = \{20\mathbf{i} - 40\mathbf{j} - 75\mathbf{k}\}$ lb, determine the *x*, *y*, *z* components of reaction at *A* and the tension in the cable.



$$\mathbf{F}_{BC} = -\frac{12}{14}F_{BC}\mathbf{i} + \frac{4}{14}F_{BC}\mathbf{j} + \frac{6}{14}F_{BC}\mathbf{k}$$

$\Sigma F_x = 0;\qquad A_x + 20 - \frac{12}{14}F_{BC} = 0$

$\Sigma F_y = 0;\qquad A_y - 40 + \frac{4}{14}F_{BC} = 0$

$\Sigma F_z = 0;\qquad -75 + \frac{6}{14}F_{BC} = 0$

$F_{BC} = 175$ lb **Ans**

$A_x = 130$ lb **Ans**

$A_y = -10$ lb **Ans**

$\Sigma M_x = 0;\qquad \frac{6}{14}(175)(4) + M_{Ax} = 0$

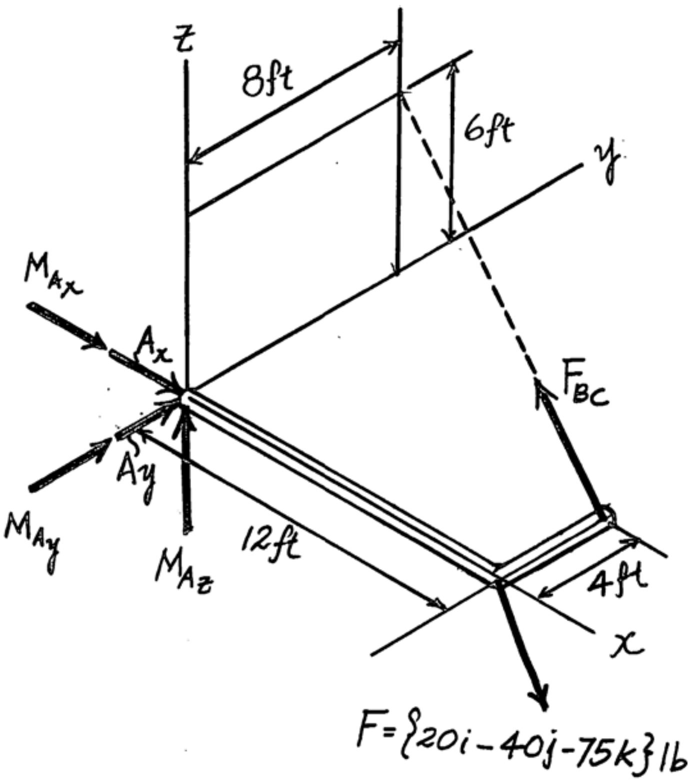
$M_{Ax} = -300$ lb·ft **Ans**

$\Sigma M_y = 0;\qquad 75(12) - \frac{6}{14}(175)(12) + M_{Ay} = 0$

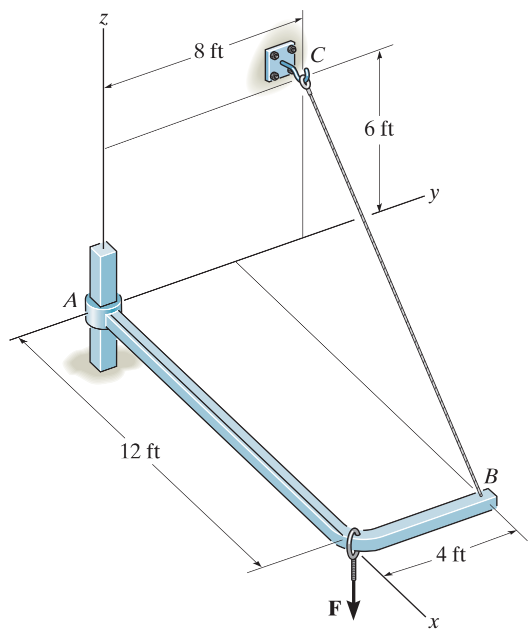
$M_{Ay} = 0$ **Ans**

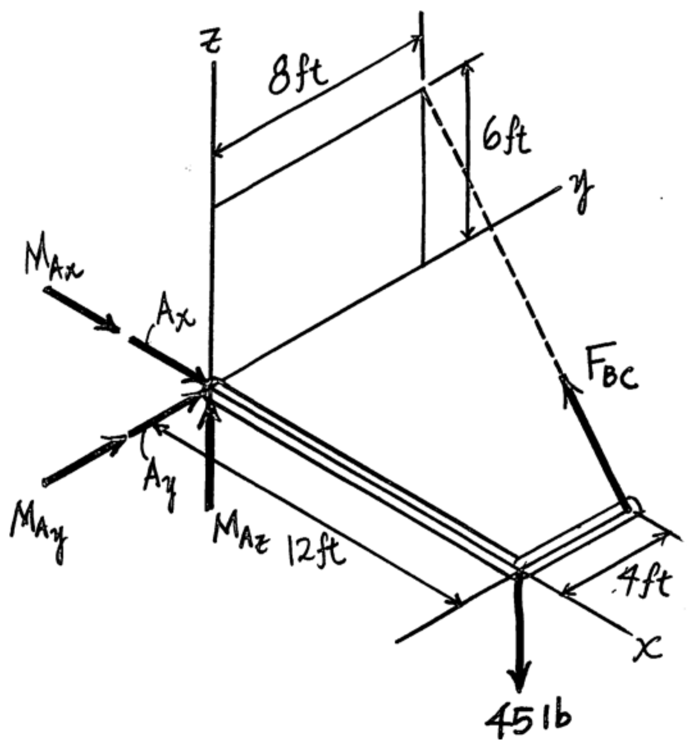
$\Sigma M_z = 0;\qquad -40(12) + \frac{12}{14}(175)(4) + \frac{4}{14}(175)(12) + M_{Az} = 0$

$M_{Az} = -720$ lb·ft **Ans**



5–83. Member *AB* is supported at *B* by a cable and at *A* by a smooth fixed *square* rod which fits loosely through the square hole of the collar. Determine the tension in cable *BC* if the force $\mathbf{F} = \{-45\mathbf{k}\}$ lb.



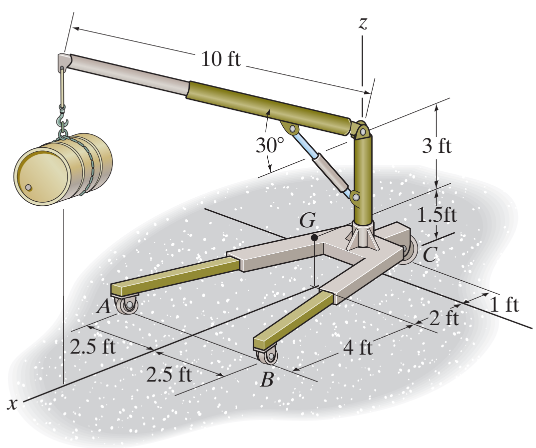


$\mathbf{F}_{BC} = -\frac{12}{14}F_{BC}\mathbf{i} +$

$\Sigma F_z = 0;\qquad \frac{6}{14}F_{BC} - 45 = 0$

$F_{BC} = 105 \text{ lb}$ **Ans**

***5–84.** Determine the largest weight of the oil drum that the floor crane can support without overturning. Also, what are the vertical reactions at the smooth wheels *A*, *B*, and *C* for this case. The floor crane has a weight of 300 lb, with its center of gravity located at *G*.



Equations of Equilibrium: The floor crane has a tendency to overturn about the y' axis, as shown on the free - body diagram in Fig. *a*. When the crane is about to overturn, the wheel at *C* loses contact with the ground. Thus

$$N_C = 0 \qquad \textbf{Ans.}$$

Applying the moment equation of equilibrium about the y' axis,

$$\Sigma M_{y'} = 0; \qquad W(10\cos 30° - 2 - 4) - 300(4) = 0$$

$$W = 451.08 \text{ lb} = 451 \text{ lb} \qquad \textbf{Ans.}$$

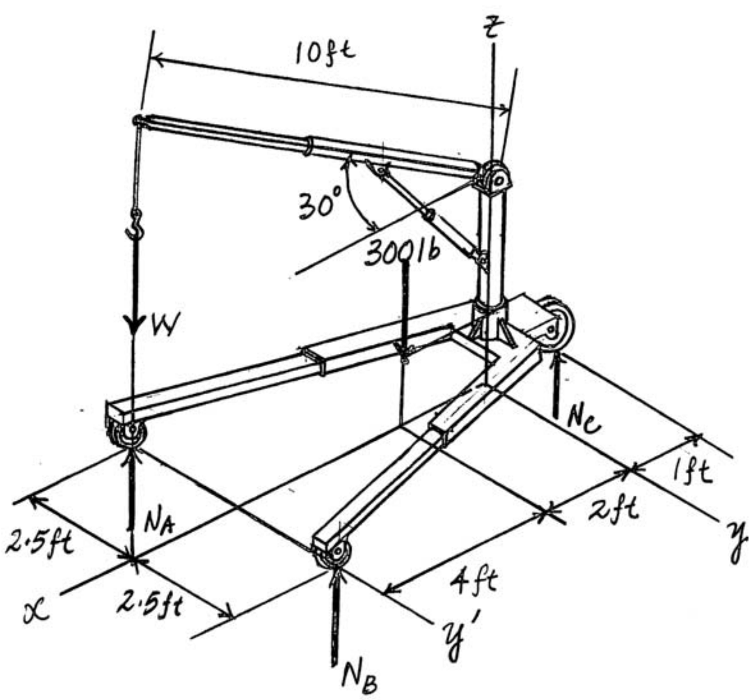
Using this result and writing the moment equation of equilibrium about the *x* axis and the force equation of equilibrium along the *z* axis,

$$\Sigma M_x = 0; \quad N_B(2.5) - N_A(2.5) = 0 \qquad (1)$$

$$\Sigma F_z = 0; \quad N_A + N_B - 300 - 451.08 = 0 \qquad (2)$$

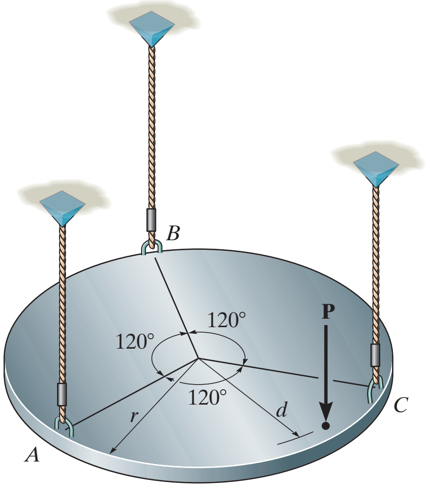
Solving Eqs. (1) and (2), yields

$$N_A = N_B = 375.54 \text{ lb} = 376 \text{ lb} \qquad \textbf{Ans.}$$



(a)

•5–85. The circular plate has a weight W and center of gravity at its center. If it is supported by three vertical cords tied to its edge, determine the largest distance d from the center to where any vertical force $\mathbf{P}$ can be applied so as not to cause the force in any one of the cables to become zero.



Assume $T_A = T_B = 0$

$\Sigma M_{a-a} = 0;\quad T_C(r + r\cos 60°) - W(r\cos 60°) - P(d + r\cos 60°) = 0$

$\Sigma F_z = 0;\quad T_C - W - P = 0$

Eliminating T_C we get

$$Wr + Pr - Pd = 0$$

$$d = r\left(1 + \frac{W}{P}\right)$$

Assume $T_C = 0$

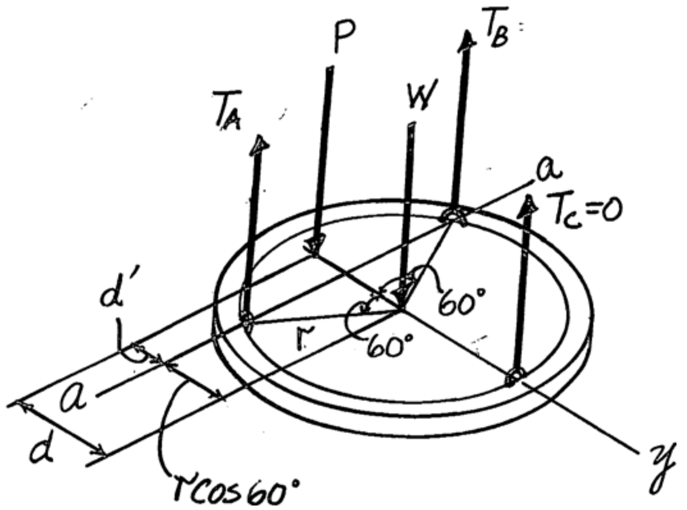
$\Sigma M_{a-a} = 0;\quad W(r\cos 60°) - P(d') = 0$

$$d' = \frac{Wr}{2P}$$

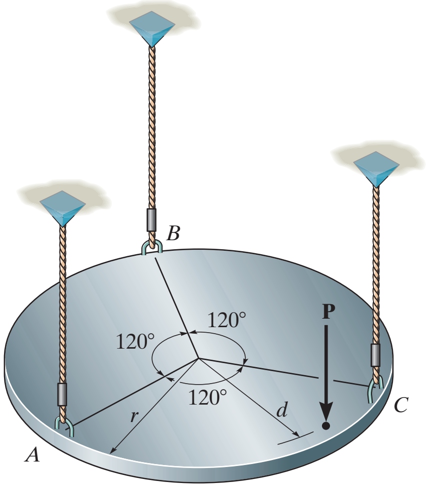
Thus

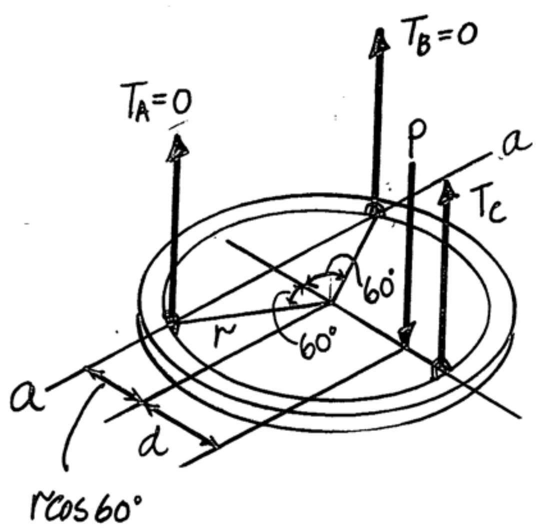
$$d = r\cos 60° + \frac{Wr}{2P}$$

$$d = \frac{r}{2}\left(1 + \frac{W}{P}\right) \quad \text{Ans}$$



5–86. Solve Prob. 5–85 if the plate's weight W is neglected.





Assume $T_A = T_B = 0$

$\Sigma M_{a-a} = 0;\quad T_C(r + r\cos 60°) - P(d + r\cos 60°) = 0$

$\Sigma F_z = 0;\quad T_C - P = 0$

$d = r$

Assume $T_C = 0$:

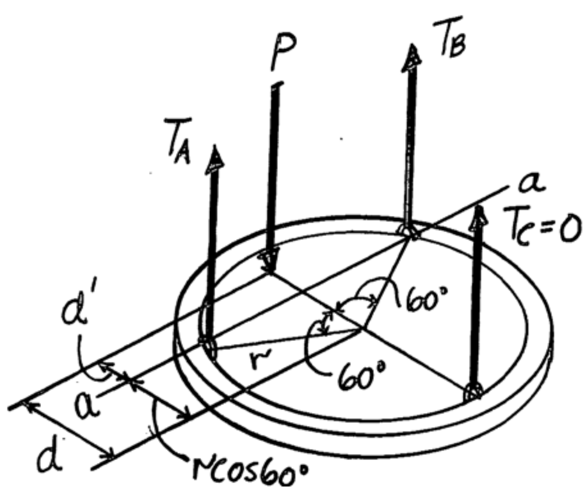
$\Sigma M_{a-a} = 0;\quad P(d') = 0$

$d' = 0$

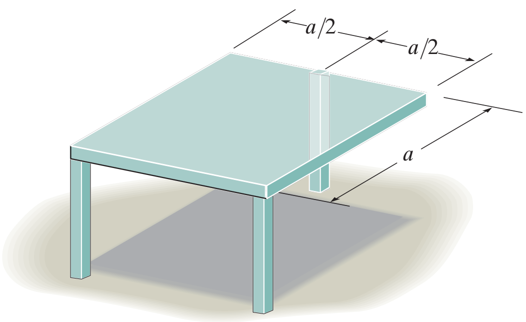
Thus,

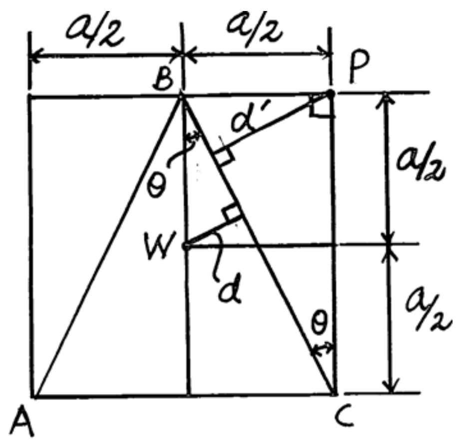
$d = r\cos 60° + 0$

$d = \frac{r}{2}$ **Ans**



5–87. A uniform square table having a weight W and sides a is supported by three vertical legs. Determine the smallest vertical force **P** that can be applied to its top that will cause it to tip over.





$$\theta = \tan^{-1}\left(\frac{\frac{a}{2}}{a}\right) = 26.565^\circ$$

$$d = \left(\frac{a}{2}\right)\sin 26.565^\circ = 0.2236\,a < \frac{a}{2}$$

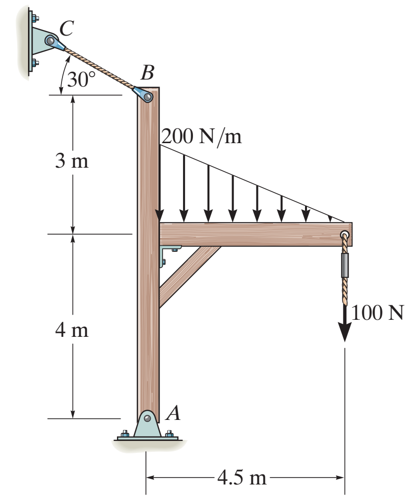
$$d' = a \sin 26.565^\circ = 0.4472\,a$$

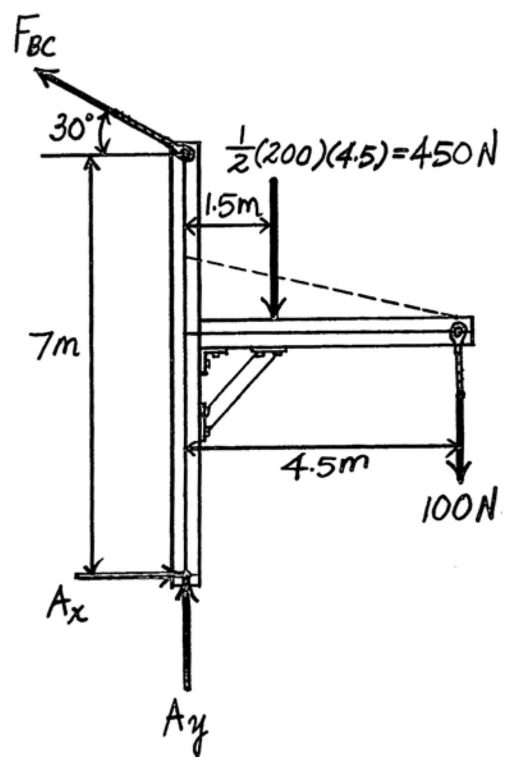
For P_{min}, put P at the corner as shown.

$$\Sigma M_{BC} = 0; \quad W\,(0.2236\,a) - P\,(0.4472\,a) = 0$$

$$P = 0.5\,W \qquad \textbf{Ans}$$

***5–88.** Determine the horizontal and vertical components of reaction at the pin A and the force in the cable BC. Neglect the thickness of the members.





$\curvearrowleft + \Sigma M_A = 0;$ $F_{BC}\cos 30°(7) - 450(1.5) - 100(4.5) = 0$

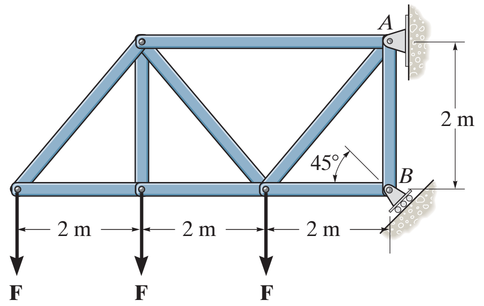
$F_{BC} = 185.58 \text{ N} = 186 \text{ N}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;$ $A_x - 185.58\cos 30° = 0$ $A_x = 161 \text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $A_y + 185.58\sin 30° - 450 - 100 = 0$

$A_y = 457 \text{ N}$ **Ans**

•5–89. Determine the horizontal and vertical components of reaction at the pin A and the reaction at the roller B required to support the truss. Set $F = 600$ N.

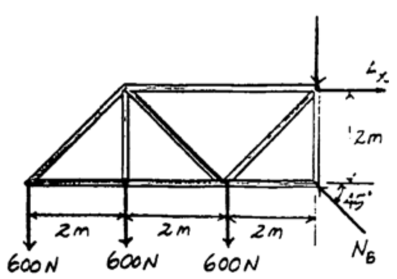


Equations of Equilibrium : The normal reaction N_B can be obtained directly by summing moments about point A.

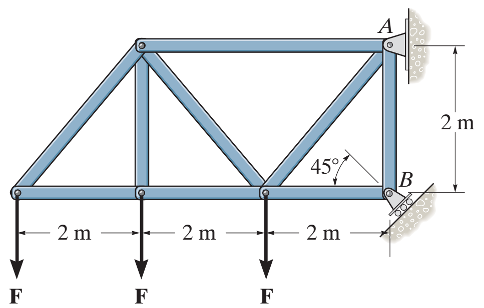
$$\curvearrowleft + \Sigma M_A = 0; \quad 600(6) + 600(4) + 600(2) - N_B \cos 45°(2) = 0$$
$$N_B = 5091.17 \text{ N} = 5.09 \text{ kN} \qquad \textbf{Ans}$$

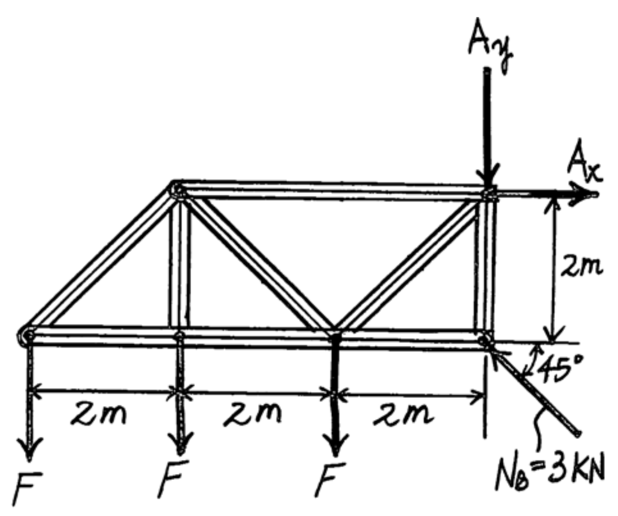
$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 5091.17\cos 45° = 0$$
$$A_x = 3600 \text{ N} = 3.60 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad 5091.17\sin 45° - 3(600) - A_y = 0$$
$$A_y = 1800 \text{ N} = 1.80 \text{ kN} \qquad \textbf{Ans}$$



5–90. If the roller at B can sustain a maximum load of 3 kN, determine the largest magnitude of each of the three forces **F** that can be supported by the truss.

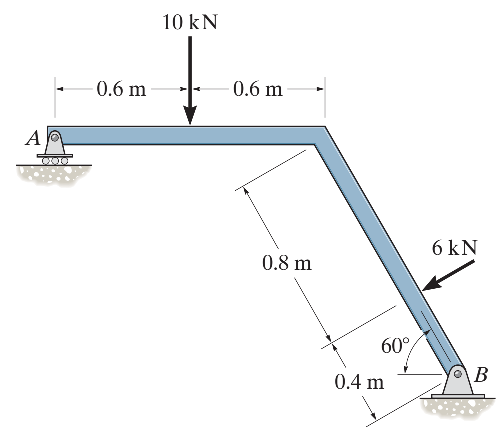




Equations of Equilibrium : The unknowns A_x and A_y can be eliminated by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \quad F(6) + F(4) + F(2) - 3\cos 45°(2) = 0$$
$$F = 0.3536 \text{ kN} = 354\text{N} \qquad \textbf{Ans}$$

5–91. Determine the normal reaction at the roller *A* and horizontal and vertical components at pin *B* for equilibrium of the member.



Equations of Equilibrium : The normal reaction N_A can be obtained directly by summing moments about point *B*.

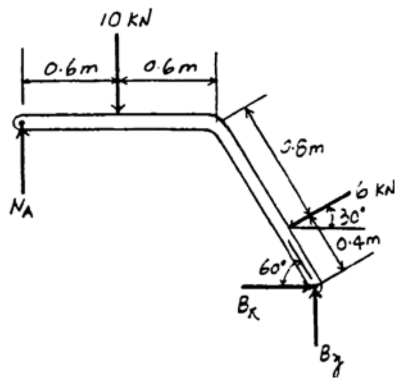
$$\circlearrowleft + \Sigma M_A = 0; \quad 10(0.6 + 1.2\cos 60°) + 6(0.4) - N_A(1.2 + 1.2\cos 60°) = 0$$

$$N_A = 8.00 \text{ kN} \qquad \textbf{Ans}$$

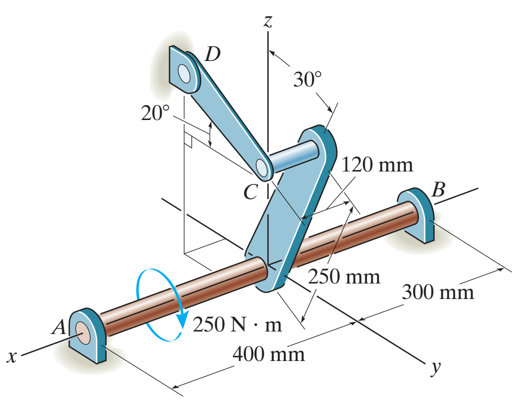
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x - 6\cos 30° = 0 \quad B_x = 5.20 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad B_y + 8.00 - 6\sin 30° - 10 = 0$$

$$B_y = 5.00 \text{ kN} \qquad \textbf{Ans}$$



***5–92.** The shaft assembly is supported by two smooth journal bearings *A* and *B* and a short link *DC*. If a couple moment is applied to the shaft as shown, determine the components of force reaction at the journal bearings and the force in the link. The link lies in a plane parallel to the *y*–*z* plane and the bearings are properly aligned on the shaft.



$\Sigma M_x = 0;\quad -250 + F_{CD}\cos 20°(0.25\cos 30°) + F_{CD}\sin 20°(0.25\sin 30°) = 0$

$F_{CD} = 1015.43\text{ N} = 1.02\text{ kN}$ **Ans**

$\Sigma(M_B)_y = 0;\quad -A_z(0.7) - 1015.43\sin 20°(0.42) = 0$

$A_z = -208.38 = -208\text{ N}$ **Ans**

$\Sigma F_z = 0;\quad -208.38 + 1015.43\sin 20° + B_z = 0$

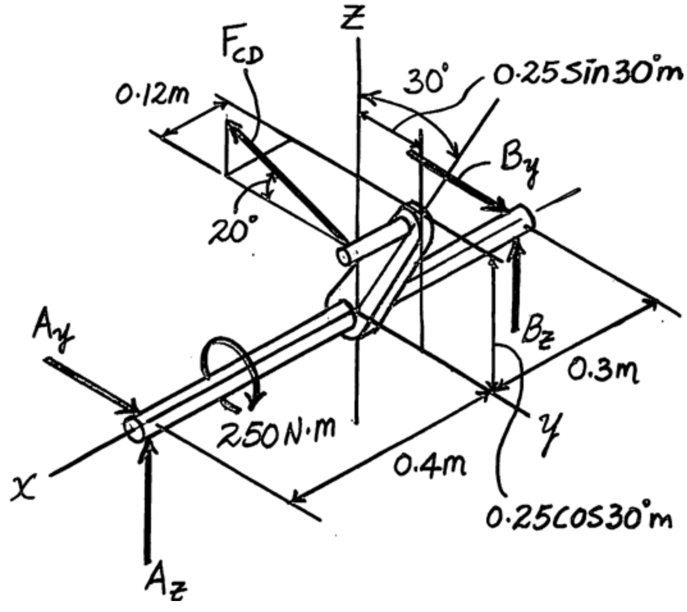
$B_z = -139\text{ N}$ **Ans**

$\Sigma(M_B)_z = 0;\quad A_y(0.7) - 1015.43\cos 20°(0.42) = 0$

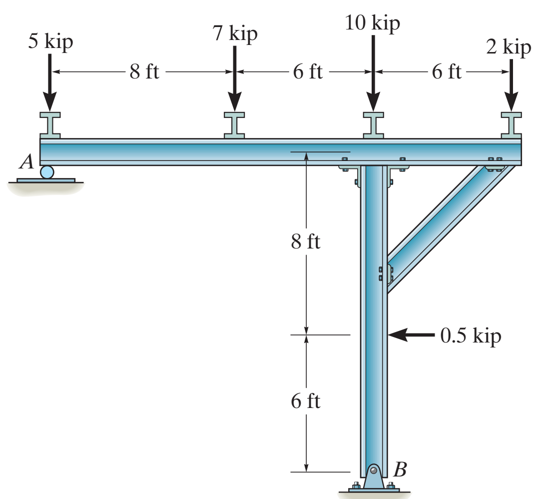
$A_y = 572.51 = 573\text{ N}$ **Ans**

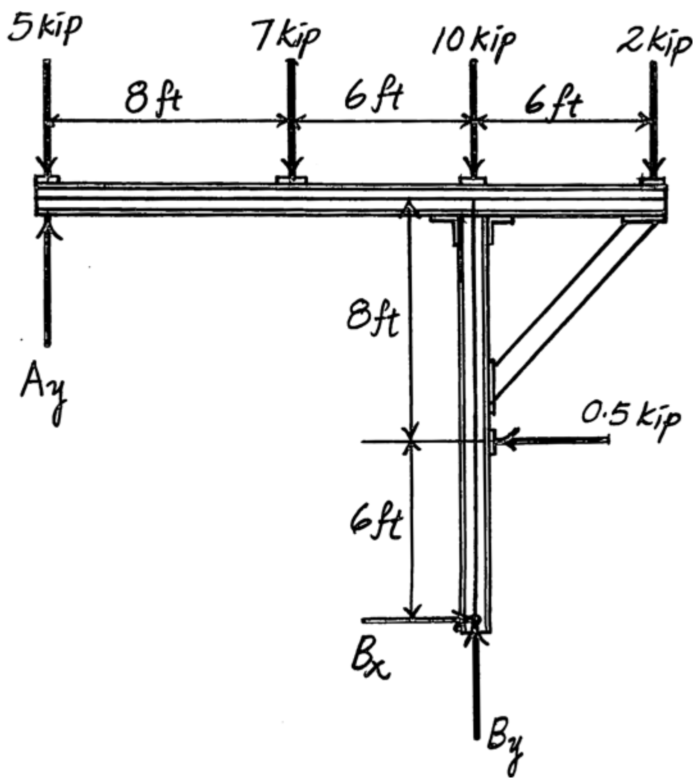
$\Sigma F_y = 0;\quad 572.51 - 1015.43\cos 20° + B_y = 0$

$B_y = 382\text{ N}$ **Ans**



•5–93. Determine the reactions at the supports *A* and *B* of the frame.





$\circlearrowleft + \Sigma M_B = 0;\quad 5(14) + 7(6) + 0.5(6) - 2(6) - A_y(14) = 0$

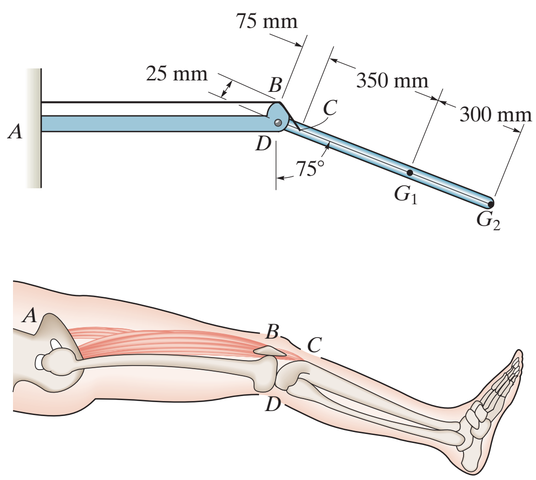
$A_y = 7.357 \text{ kip} = 7.36 \text{ kip}$ **Ans**

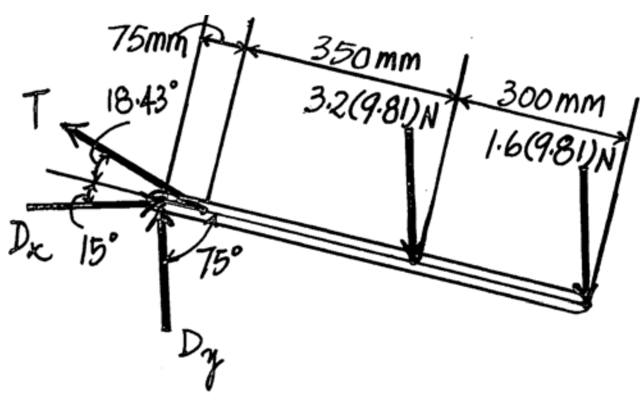
$\xrightarrow{+} \Sigma F_x = 0;\quad B_x - 0.5 = 0 \qquad B_x = 0.5 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad B_y + 7.357 - 5 - 7 - 10 - 2 = 0$

$B_y = 16.6 \text{ kip}$ **Ans**

5–94. A skeletal diagram of the lower leg is shown in the lower figure. Here it can be noted that this portion of the leg is lifted by the quadriceps muscle attached to the hip at A and to the patella bone at B. This bone slides freely over cartilage at the knee joint. The quadriceps is further extended and attached to the tibia at C. Using the mechanical system shown in the upper figure to model the lower leg, determine the tension in the quadriceps at C and the magnitude of the resultant force at the femur (pin), D, in order to hold the lower leg in the position shown. The lower leg has a mass of 3.2 kg and a mass center at G_1; the foot has a mass of 1.6 kg and a mass center at G_2.





$\curvearrowleft + \Sigma M_D = 0;\quad T\sin 18.43°(75) - 3.2(9.81)(425\sin 75°)$

$- 1.6(9.81)(725\sin 75°) = 0$

$T = 1006.82\text{ N} = 1.01\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad D_y + 1006.82\sin 33.43° - 3.2(9.81) - 1.6(9.81) = 0$

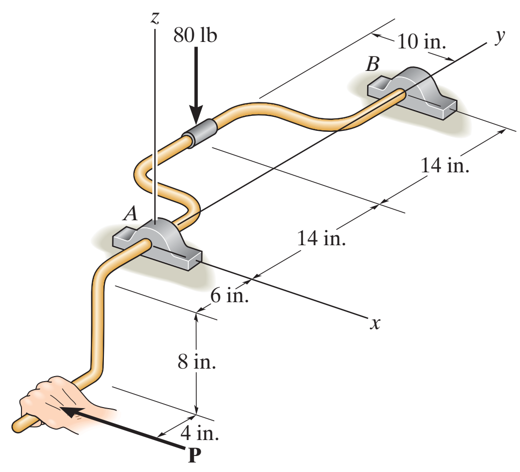
$D_y = -507.66\text{ N}$

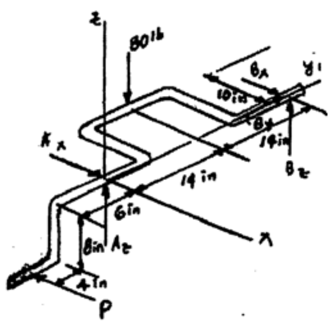
$\xrightarrow{+} \Sigma F_x = 0;\quad D_x - 1006.82\cos 33.43° = 0$

$D_x = 840.20\text{ N}$

$$F_D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-507.66)^2 + 840.20^2} = 982\text{ N}$$ **Ans**

5–95. A vertical force of 80 lb acts on the crankshaft. Determine the horizontal equilibrium force P that must be applied to the handle and the x, y, z components of force at the smooth journal bearing A and the thrust bearing B. The bearings are properly aligned and exert only force reactions on the shaft.





$\Sigma M_y = 0;\quad P(8) - 80(10) = 0 \qquad P = 100\text{ lb}$ **Ans**

$\Sigma M_x = 0;\quad B_z(28) - 80(14) = 0 \qquad B_z = 40\text{ lb}$ **Ans**

$\Sigma M_z = 0;\quad -B_x(28) - 100(10) = 0 \qquad B_x = -35.7\text{ lb}$ **Ans**

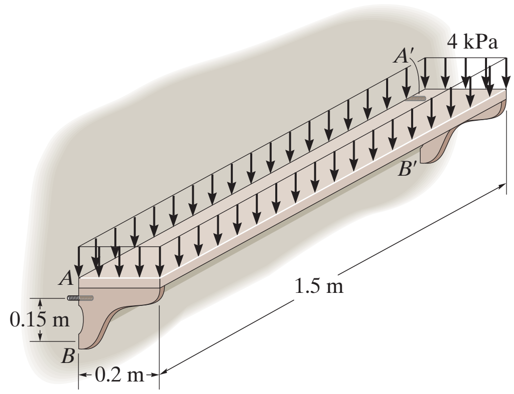
$\Sigma F_x = 0;\quad A_x + (-35.7) - 100 = 0 \qquad A_x = 136\text{ lb}$ **Ans**

$\Sigma F_y = 0;\quad B_y = 0$ **Ans**

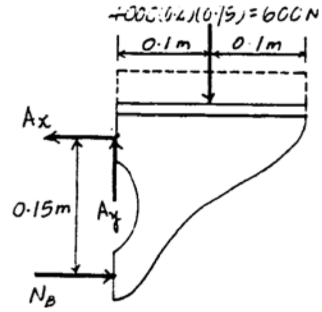
$\Sigma F_z = 0;\quad A_z + 40 - 80 = 0 \qquad A_z = 40\text{ lb}$ **Ans**

Negative sign indicates that B_x acts in the opposite sense to that shown on the FBD.

***5–96.** The symmetrical shelf is subjected to a uniform load of 4 kPa. Support is provided by a bolt (or pin) located at each end A and A' and by the symmetrical brace arms, which bear against the smooth wall on both sides at B and B'. Determine the force resisted by each bolt at the wall and the normal force at B for equilibrium.



Equations of Equilibrium : Each shelf's post at its end supports half of the applied load, ie, $4000(0.2)(0.75) = 600$ N. The normal reaction N_B can be obtained directly by summing moments about point A.



$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(0.15) - 600(0.1) = 0 \quad N_B = 400 \text{ N} \quad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 400 - A_x = 0 \quad A_x = 400\text{N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 600 = 0 \quad A_y = 600 \text{ N}$$

The force resisted by the bolt at A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{400^2 + 600^2} = 721 \text{ N} \quad \textbf{Ans}$$